RESTRICTED NEYMAN-PEARSON APPROACH BASED SPECTRUM SENSING IN COGNITIVE RADIO SYSTEMS

A THESIS

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MASTER OF SCIENCE

By

Esma Turgut

June 2012

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Sinan Gezici (Supervisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Defne Aktaş

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. İbrahim Körpeoğlu

Approved for the Graduate School of Engineering and Sciences:

Prof. Dr. Levent Onural
Director of Graduate School of Engineering and Sciences

ABSTRACT

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Esma Turgut
M.S. in Electrical and Electronics Engineering
Supervisor: Assist. Prof. Dr. Sinan Gezici
June 2012

Over the past decade, the demand for wireless technologies has increased enormously, which leads to a perceived scarcity of the frequency spectrum. Meanwhile, static allocation of the frequency spectrum leads to under-utilization of the spectral resources. Therefore, dynamic spectrum access has become a necessity. Cognitive radio has emerged as a key technology to solve the conflicts between spectrum scarcity and spectrum under-utilization. It is an intelligent wireless communication system that is aware of its operating environment and can adjust its parameters in order to allow unlicensed (secondary) users to access and communicate over the frequency bands assigned to licensed (primary) users when they are inactive. Therefore, cognitive radio requires reliable spectrum sensing techniques in order to avoid interference to primary users. In this thesis, the spectrum sensing problem in cognitive radio is studied. Specifically, the restricted Neyman-Pearson (NP) approach, which maximizes the average detection probability under the constraints on the minimum detection and false alarm probabilities, is applied to the spectrum sensing problem in cognitive radio systems in the presence of uncertainty in the prior probability distribution of primary users' signals. First, we study this problem in the presence of Gaussian noise and assume that primary users' signals are Gaussian. Then, the problem is reconsidered for non-Gaussian noise channels. Simulation results are obtained in order to compare the performance of the restricted NP approach with the existing methods such as the generalized likelihood ratio test (GLRT) and energy detection. The restricted NP approach outperforms energy detection in all cases. It is also shown that the restricted NP approach can provide important advantages over the GLRT in terms of the worst-case detection probability, and sometimes in terms of the average detection probability depending on the situation in the presence of imperfect prior information for Gaussian mixture noise channels.

Keywords: Cognitive radio, detection, spectrum sensing, Neyman-Pearson.

ÖZET

BİLİŞSEL RADYO SİSTEMLERİNDE KISITLI NEYMAN-PEARSON YAKLAŞIMI TABANLI SPEKTRUM SEZME

Esma Turgut
Elektrik ve Elektronik Mühendisliği Bölümü Yüksek Lisans
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Geçtiğimiz on yıl içinde kablosuz teknolojilere olan talep, frekans spektrumunda farkedilebilir bir kıtlığa yol açacak şekilde fazlasıyla artmıştır. Aynı zamanda, frekans spektrumunun statik tahsisi spektrumun verimsiz kullanımına yol açmıştır. Dolayısıyla dinamik spektrum erişimi bir zorunluluk haline gelmiştir. Bilişsel radyo, spektrum kıtlığı ve spektrumun verimsiz kullanımı arasındaki çatışmaları çözmek için anahtar teknoloji olarak ortaya çıkmıştır. Bilişsel radyo lisanssız (ikincil) kullanıcıların lisanslı (birincil) kullanıcılara ayrılan frekans bandlarına onlar aktif değilken erişip iletişim kurabilmelerine izin vermek için parametrelerini ayarlayabilen ve çalışma ortamının farkında olan, akıllı bir kablosuz iletişim sistemidir. Bilişsel radyo, birincil kullanıcılarla girişimi engellemek amacıyla güvenilir spektrum sezme tekniklerine ihtiyaç duyar. Bu tezde, bilişsel radyolardaki spektrum sezme konusu çalışılmaktadır. Özel olarak, minimum tespit ve yanlış alarm olasılıkları üzerindeki kısıtlamalar altında ortalama tespit olasılığını maksimuma çıkaran kısıtlı Neyman-Pearson (NP) yaklaşımı, birincil kullanıcıların sinyallerinin önsel olasılık dağılımındaki belirsizliğin varlığında bilişsel radyolardaki spektrum sezme problemine uygulanmaktadır. İlk olarak Gauss gürültü varlığında bu problem incelenmekte ve birincil kullanıcıların sinyallerinin Gauss dağılımı olduğu varsayılmaktadır. Daha sonra, Gauss olmayan gürültü kanalları için problem tekrar ele alınmaktadır. Kısıtlı NP yaklaşımının performansını, genelleştirilmiş olabilirlik oranı testi (GLRT) ve enerji algılama gibi varolan metodların performansıyla karşılaştırmak için simülasyon sonuçları elde edilmektedir. Her iki gürültü kanalında da kısıtlı NP yaklaşımı enerji algılama metodunu geride bırakmaktadır. Ayrıca, Gauss karışım gürültü kanalları için önsel bilgi eksikliğinin varlığında kısıtlı NP yaklaşımı, en kötü durumdaki tespit olasılığı açısından ve bazen de duruma bağlı olarak ortalama tespit olasılığı açısından GLRT üzerinde önemli avantajlar sağlayabilmektedir.

Anahtar Kelimeler: Bilişsel radyo, sezim, spektrum sezme, Neyman-Pearson.

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Dedicated to my husband Emrah . . .

Chapter 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

In the past decade, wireless technologies have grown rapidly and the spectrum resources have faced difficulties in meeting the increasing demand [5]. Since spectrum is an unexpandable natural resource, increasing wireless services such as mobile phones, 3G and 4G mobile services, wireless internet and many others lead to spectrum scarcity [5]. In traditional spectrum allocation, frequency bands are assigned to specific licensed users and other users cannot use those bands even if the licensed users are idle. The National Telecommunication and Information Administration's (NTIA) chart of spectrum frequency allocations in Figure 1.1 indicates that within the current spectrum regulatory framework, all of the frequency bands are exclusively allocated to specific services and the Federal Communications Commission (FCC) does not allow violation from unlicensed users because of its regulations. However, actual measurements of spectrum utilization show that many assigned bands are largely unoccupied most of the time.

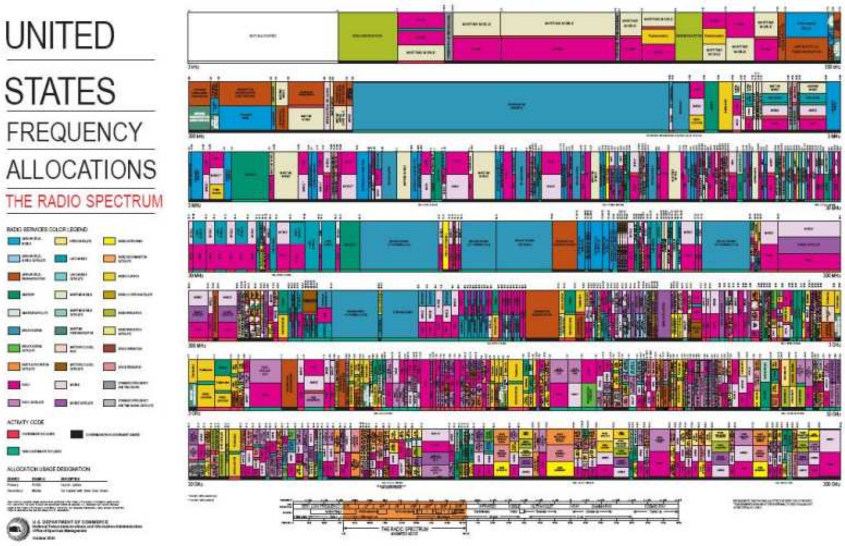
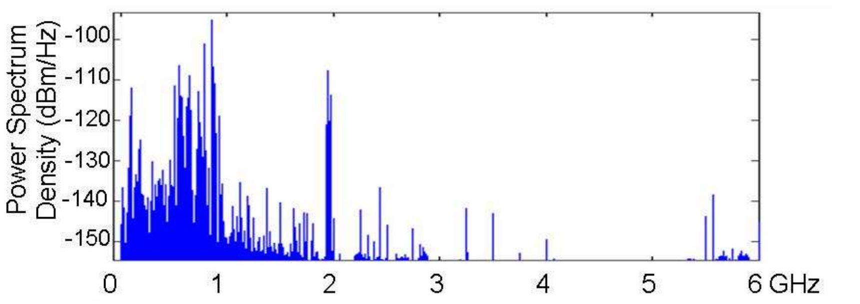


Figure 1.1: The NTIA's frequency allocation chart [1].

Figure 1.2 shows the measurement of 0-6 GHz spectrum utilization taken by the Berkeley Wireless Research Center (BWRC) in downtown Berkeley, CA. As can be seen, spectrum is utilized more intensely at frequencies below 3 GHz, while spectrum is under-utilized in 3-6 GHz bands [2]. Moreover, measurements taken over 10 minutes in the same Berkeley location (Figure 1.3) indicate that there are also temporal gaps in the spectrum usage even in the 0 to 2.5 GHz band, which is considered to be very crowded. Recent measurements taken in the US and the world have shown similar results. These measurements lead to the serious questioning of the convenience of the current regulatory regime and possibly provide the opportunity to solve the spectrum scarcity problem. According to a recent report by the USA Federal Communications Commission (FCC), fixed spectrum allocation leads to utilization of spectrum very inefficiently in almost all currently deployed frequency bands and, they therefore recommend allowing the secondary users to fill the available spectrum holes [6].

Cognitive radio, first proposed in 1999 in [7], has emerged as a promising approach to solve the conflicts between spectrum under-utilization and spectrum scarcity. In other words, cognitive radio was proposed to improve spectrum utilization via opportunistic spectrum sharing. The basic idea behind the cognitive



Freq (GHz)	0~1	1~2	2~3	3~4	4~5	5~6
Utilization(%)	54.4	35.1	7.6	0.25	0.128	4.6

Figure 1.2: Measurement of 0-6 GHz spectrum utilization in downtown Berkeley, CA [2].

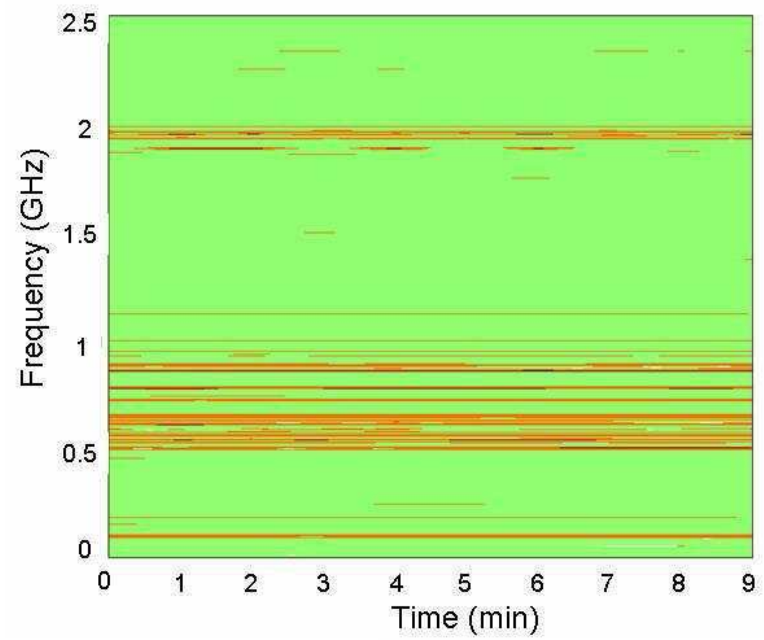


Figure 1.3: Temporal variation of the spectrum utilization (0-2.5 GHz) in downtown Berkeley, CA [2].

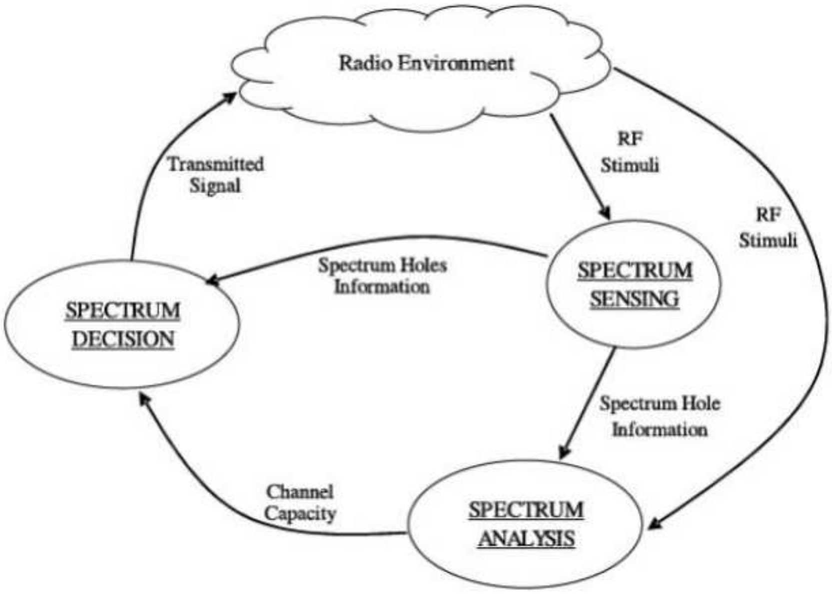


Figure 1.4: Cognitive cycle [3].

radio is allowing unlicensed (secondary) users to access and communicate over the frequency bands assigned to licensed (primary) users when they are inactive. Hence, a cognitive radio may be defined as an intelligent wireless communication system that is aware of its environment and can change its transmission and reception parameters based on interaction with the environment in which it operates in order to have reliable communication without interfering with the primary users and optimize spectrum usage.

Cognitive radio has two main characteristics: cognitive capability and reconfigurability [3]. Firstly, cognitive capability refers to the ability of the radio technology to capture or sense the information from its radio environment. This ability is useful to identify the spectrum holes at a specific time or location. To do this cognitive radio has to perform some tasks shown in Figure 1.4, which is referred as the cognitive cycle. The cognitive cycle has three main steps: spectrum sensing, spectrum analysis, and spectrum decision. Secondly, reconfigurability means adjusting operating parameters according to radio environment. There are several reconfigurable parameters such as operating frequency, modulation, transmission power, and communication technology.

In the next the section spectrum sensing task is discussed in detail.

1.2 Spectrum Sensing

In cognitive radio systems, one of the most important tasks is spectrum sensing, i.e., detection of the presence or absence of primary users. Reliable and fast spectrum sensing is important because secondary users should cause little interference to primary users while achieving higher utilization of spectrum holes. Spectrum sensing is a very challenging task due to several reasons [8]. One of them is low signal to noise ratio (SNR). For example, if the transmitted signal of a primary user is in an deep fade, then detection becomes very hard for a secondary user even if the primary and secondary users are very close to each other. A practical scenario is that the received SNR is less than -20 dB for a secondary user several hundred meters away from a wireless microphone which transmits with a power less than 50 mW and a bandwidth less than 200 kHz [8]. Secondly, multipath fading and time dispersion of the wireless channels are the other problems which make sensing complicated. First one may cause great fluctuations in signal power and second one may make coherent detection unreliable [8]. Thirdly, noise power uncertainty is another problem that makes spectrum sensing a challenging task and it has been heavily studied in recent years such as in [9], [10], [11].

The spectrum sensing problem has been studied extensively and it has become a very active research area in recent years. Many sensing methods have been proposed to identify the presence of signal transmission. Sensing methods can be classified into three categories according to their requirements [8]:

(1) Methods requiring both source signal and noise power information (non blind detection)

(2) Methods requiring only noise power information (semi blind detection)

(3) Methods requiring no information on signal and noise power (totally blind detection)

The first category includes methods such as likelihood ratio test [12], matched filtering [12], [13], and cyclostationary feature detection [14], [15], [16], [17]. Energy detection [13], [12], [18] and wavelet-based sensing [19] methods belong to the second category. On the other hand, sensing techniques like eigenvalue-based sensing, covariance-based sensing and blindly combined energy detection are included in the third category.

Among all sensing techniques energy detection, matched filtering and cyclostationary detection are the most popular ones. Choice of a spectrum sensing technique depends on available information about primary signals as mentioned above. The matched filter provides the best performance among these three methods but it needs complete prior knowledge about the primary user signal. If the primary user signal exhibits certain periodicity in the mean and autocorrelation, then cognitive radio uses cyclostationary feature detection. On the other hand, energy detection is the optimal and simplest one if there is limited information on structure of primary users' signal [20]. However, the energy detector needs to know the noise variance for proper operation. In practice, it is difficult to know noise variance accurately and uncertainty in the noise variance can degrade the performance of the energy detector dramatically.

In the following subsections, first a generic formulation of the spectrum sensing problem is provided then these three popular sensing methods are discussed in detail.

1.2.1 Spectrum Sensing Problem

Spectrum sensing is based on a well-known technique called signal detection. Signal detection is a method used to identify the presence of a signal in a noisy environment [21]. The signal detection problem can be formulated as a simple hypothesis-testing problem in which we assume that there are two possible hypotheses, $\mathcal{H}_0$ and $\mathcal{H}_1$ [22];

$$x(n) = \begin{cases} w(n), & \mathcal{H}_0 \\ s(n) + w(n), & \mathcal{H}_1 \end{cases}$$

where $x(n)$ is the received signal at the secondary user, $s(n)$ is the transmitted signal of the primary user and $w(n)$ is noise (not necessarily white Gaussian noise) of variance σ^2. $\mathcal{H}_0$ and $\mathcal{H}_1$ are the noise-only and the signal plus noise hypotheses, respectively. In other words, $\mathcal{H}_0$ declares absence the of the signal while $\mathcal{H}_1$ points out the presence of the signal. The hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are sometimes referred as the null and alternative hypotheses, respectively. There are four possible cases for the detected signal [21]:

1) Declaring $\mathcal{H}_0$ when $\mathcal{H}_0$ is true $(\mathcal{H}_0|\mathcal{H}_0)$

2) Declaring $\mathcal{H}_1$ when $\mathcal{H}_1$ is true $(\mathcal{H}_1|\mathcal{H}_1)$: Detection

3) Declaring $\mathcal{H}_0$ when $\mathcal{H}_1$ is true $(\mathcal{H}_0|\mathcal{H}_1)$: Miss detection

4) Declaring $\mathcal{H}_1$ when $\mathcal{H}_0$ is true $(\mathcal{H}_1|\mathcal{H}_0)$: False alarm

The primary aim of signal detection is to increase the probability of detection $P_d = P(\mathcal{H}_1|\mathcal{H}_1)$ and decrease the probability of false alarm $P_f = P(\mathcal{H}_1|\mathcal{H}_0)$ as much as possible. Miss detections and false alarms are important issues for spectrum sensing since the first one results in interference with primary user signals and the second one is desired to be as low as possible so that secondary users can utilize all possible transmission opportunities (i.e., can have high data rates).

1.2.2 Matched Filtering

The matched filter is the optimal detector in the stationary Gaussian noise if the primary user signal is known to the secondary user, since it maximizes the received SNR [3]. Although it is computationally simple and due to coherency it requires less time to achieve high processing gain, a matched filter requires *a priori* knowledge of the primary user signal such as modulation type and order, pulse shape, and packet format and also requires knowledge on the channel responses from the primary user to the receiver [3], [23]. If this information is not accurate, then performance of the matched filter significantly degrades. In practice, cognitive radios do not know the signal perfectly so this technique is not well-suited for spectrum sensing in cognitive radio systems.

1.2.3 Cyclostationary Feature Detection

Another spectrum sensing method is cyclostationary feature detection. Modulated signals are in general coupled with sine wave carriers, pulse trains, repeating spreading, hopping sequences, or cyclic prefixes, which result in built-in periodicity [3]. Mean and autocorrelation of these signals exhibit periodicity so they are classified as cyclostationary. We can detect these features by analyzing a spectral correlation function. On the other hand, noise is wide-sense stationary with no correlation. Hence, the spectral correlation function differentiates the noise energy from the modulated signal energy. Therefore, cyclostationary feature detectors are robust to noise uncertainty and work even in very low SNR region unlike the energy detectors [24], [25]. Hence, FCC has suggested cyclostationary detectors as a useful alternative to enhance the detection sensitivity in cognitive radio networks [6].

In cyclostationary feature detection, the period of the primary signal should be known as *a priori* knowledge. This can be possible in the early stage of a cognitive radio application, because only limited spectrum bands, such as TV bands, are open to cognitive radio users and the characteristics of the primary signals are well known to the secondary users. When CRs are allowed to use wide-band spectrum in the future, the periods of some modulated primary signals may not be known by the secondary users. In such a situation, too much effort is needed to search for cyclic frequencies which means huge computational complexity and the loss of the ability to differentiate the primary signal from the interference that is also cyclostationary [25]. Also, this type detection can only be applicable for few primary signals with such characteristics.

1.2.4 Energy Detection

If only the noise power is known to the receiver, the optimal detector is the energy detector [20]. The detection mechanism of an energy detector is simple as depicted in Figure 1.5. The detector first computes the signal energy and then compares it to a predetermined threshold value to decide whether the primary signal is present or not. This threshold is set according to the desired probability of false alarm. Since it is easy to implement and less expensive when compared to other methods, the recent work on detection of the primary user has generally adopted the energy detector [26], [20]. One drawback of the energy detector is that it is very sensitive to uncertainty in the noise power. If the noise power is not perfectly known, the energy detector performs poorly. Another one is that the energy detector can only determine the presence of the signal not its type or source. Hence, false alarm probability can increase due to unintended signals.

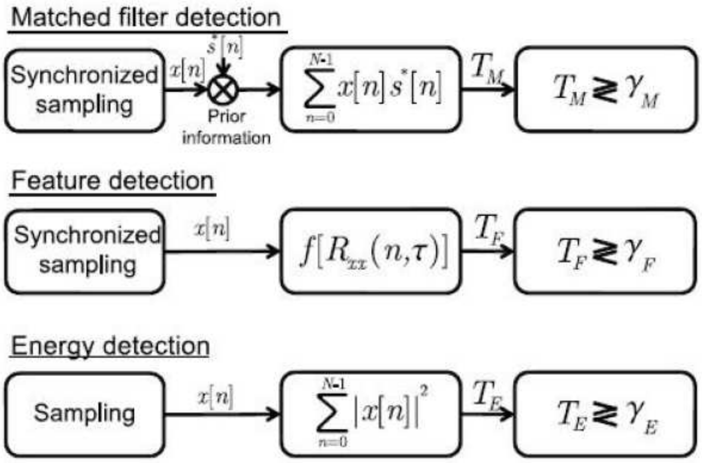


Figure 1.5: Three main types of spectrum sensing techniques [4].

In practice, energy detection is especially appropriate for wide-band spectrum sensing. Wide-band spectrum sensing is usually done in two steps. In the first step, low-complexity energy detection is applied to search for possible idle sub bands and then more advanced spectrum sensing techniques such as cyclostationary feature detection are employed to determine whether sub band candidates are available or not for secondary usage [25].

1.3 Thesis Organization and Contributions

In this thesis, we study the spectrum sensing problem in cognitive radio systems. Our main contribution is developing an application of the restricted NP algorithm to the spectrum sensing problem in cognitive radios. The important point is that the restricted NP approach assumes neither the perfect knowledge of the prior information nor the absence of it. This is usually the case in cognitive radios, i.e., some information about the prior distribution is available in many cases but that information is not perfect most of the time. Therefore, we take into account the uncertainty in the prior distribution of primary signals available to cognitive radio using the restricted NP approach in spectrum sensing. The employed restricted NP approach maximizes the average detection probability while the minimum detection probability under the constraints that the minimum detection probability should be larger than a predefined value and that the false-alarm probability should be less than a significance level.

The rest of the thesis is organized as follows:

In Chapter 2, we first provide background information about common hypothesis testing techniques, and mention the techniques which take into account partial prior information. Second, the description of the restricted NP approach is provided [27]. Then, we explain how the optimal decision rule is found. Finally, a three step algorithm for the restricted NP approach is presented.

In Chapter 3, we investigate how the generic restricted NP approach can be applied to the spectrum sensing problem in cognitive radio systems. First, Gaussian noise channels are considered. In the presence of Gaussian noise, first Gaussian primary user signals are taken into consideration. Then, we reconsider the problem when no special distributions for primary user signals are assumed. We continue this chapter by presenting applications of the restricted NP approach to the spectrum sensing problem over non-Gaussian channels. For each scheme, we also provide the formulation of the GLRT and energy detection approaches in order to evaluate the performance of the restricted NP approach.

In Chapter 4, numerical results for the restricted NP method, and existing GLRT and energy detection approaches are provided. We present simulation results for various scenarios first in the presence of Gaussian noise, then in the presence of Gaussian mixture noise, respectively.

In Chapter 5, the main ideas of the thesis are summarized and some possible future works are proposed to extend this research.

Chapter 2

Restricted Neyman-Pearson Approach

2.1 Background Information

As mentioned in the previous section, spectrum sensing is one of the most important tasks in cognitive radio systems. That is, secondary users should detect the presence of primary users in order to avoid interference and determine spectrum holes. Since spectrum sensing is based on signal detection, it is important to apply the optimal detection rule. For any given decision problem, there are a number of possible decision strategies that can be applied. Bayesian, minimax and Neyman Pearson hypothesis testings are three most common approaches for the formulation of testing [22]. The Bayesian approach assumes that perfect prior knowledge is available whereas the minimax approach completely ignores the prior information [28]. Therefore, Bayesian and minimax decision rules are the two extreme cases of prior information. In the Bayesian framework, all forms of uncertainty are represented by a prior probability distribution, and posterior probabilities are used to make a decision. If the prior probabilities are unknown, then the Bayesian approach is not appropriate since it is unlikely that a single decision rule would minimize the average risk for every possible prior distribution. Hence, in this case the minimax approach is an alternative decision rule which minimizes the maximum of risk functions defined over the parameter space [22]. In practice, complete prior information is available in rare situations and usually only partial prior information is accessible [29], [30]. Absence of exact prior information causes degradation in the performance of the Bayesian approach. On the other hand, ignoring the partial prior information is the primary reason for poor performance of the minimax approach. Therefore, various approaches that take partial information into account have been proposed such as [29], [30], [31], [32] and [33].

The restricted Bayes decision rule is one of the decision rules that take partial information into account. It minimizes the Bayes risk under a constraint on the individual conditional risks [34]. The value of the constraint is determined according to the amount of uncertainty in the prior information, so the restricted Bayesian approach is a compromise between the Bayesian and minimax approaches [30]. Since uncertainty measurement is necessary in the calculation of the Bayes risk and imposing a constraint on the conditional risks is a non-probabilistic description of uncertainty, the restricted Bayes approach combines probabilistic and non-probabilistic descriptions of uncertainty. In [27], the idea of the restricted Bayes approach is adopted to the NP framework, which is explained in detail in the next paragraph.

In the NP approach, the goal is to maximize the detection probability under a constraint on the false-alarm probability by deciding between the null and alternative hypotheses [22]. In some cases the null hypothesis may be composite, and the false-alarm constraint for all possible distributions can be applied in such situations [35], [36]. On the other hand, when alternative hypothesis is composite, there are various approaches to this problem. One of them is to search for a uniformly most powerful (UMP) test which maximizes the detection probability by imposing false-alarm constraints for all possible probability distributions under the alternative hypothesis. However, finding a UMP test is very exceptional in many cases [22]. Hence, other approaches have been proposed. In one approach, for example, the average detection probability is maximized under the false-alarm constraint [37], [38], [39]. In this case, a prior distribution of the parameter under the alternative hypothesis should be known to be able to calculate average detection probability. This max-mean approach is like the classical NP approach. On the other hand, the max-min approach, the aim of which is to maximize the minimum detection probability under the false-alarm constraint, can be applied if a prior distribution is not available [35], [36]. The max-min approach is conservative because its solution is a NP decision rule corresponding to the least-favorable distribution of the unknown parameter under the alternative hypothesis.

In [27], the restricted NP approach is studied which can be considered as an application of the restricted Bayes approach to the NP framework [34], [30]. This approach maximizes the average detection probability under the constraints that the minimum detection probability should be larger than a predefined value and that the false-alarm probability should be less than a significance level. Thus, the uncertainty in the prior distribution information under the alternative hypothesis is considered, and the constraint on the minimum (worst-case) detection probability is adjusted depending on the amount of uncertainty [27]. The classical NP (max-mean) and the max-min criteria are the special cases of the restricted NP criterion.

In this chapter, the restricted NP approach is presented, and then its application to the spectrum sensing problem in cognitive radio systems is provided in the next chapter.

2.2 Restricted Neyman-Pearson Approach

2.2.1 Problem Formulation

$p_\theta(\mathrm{x})$ is a family of probability densities that takes values in a parameter set Λ, where $\mathrm{x} \in \mathbb{R}^K$ represents the measurement for spectrum sensing (e.g., energy measurements). The spectrum sensing problem can be formulated as a generic binary composite hypothesis-testing problem as follows [27]:

$$\mathcal{H}_0 : \theta \in \Lambda_0$$
$$\mathcal{H}_1 : \theta \in \Lambda_1 \tag{2.1}$$

where $\mathcal{H}_i$ denotes the ith hypothesis and Λ_i is the set of possible parameter values under $\mathcal{H}_i$ for $i = 0, 1$ [22]. $\mathcal{H}_0$ and $\mathcal{H}_1$ are the null and alternative hypotheses, respectively. In other words, $\mathcal{H}_0$ declares absence of primary signal (PS) user while $\mathcal{H}_1$ points out presence of primary signal (PS) user. Parameter sets Λ_0 and Λ_1 are disjoint, and their union is the parameter space $\Lambda = \Lambda_0 \cup \Lambda_1$ [27].

The probability distributions of parameter θ under $\mathcal{H}_0$ and $\mathcal{H}_1$ are denoted by $w_0(\theta)$ and $w_1(\theta)$, respectively. It is assumed in [27] that these distributions are uncertain. For example, this uncertainty can be due to estimation errors of probability density function (PDF) estimates based on previous decisions (experience) [27]. As the estimation error increases, the amount of uncertainty also increases [27].

According to the restricted NP approach, maximization of the average detection probability, which is obtained using the uncertain PDF $w_1(\theta)$, under constraints on the worst-case detection and false-alarm probabilities is the our goal [27]. Before a mathematical formulation of the restricted NP criterion is presented, the definitions of detection and false-alarm probability of a decision rule are given in [27] as follows:

$$P_D(\phi;\theta) \triangleq \int_\Gamma \phi(\mathrm{x}) p_\theta(\mathrm{x}) d\mathrm{x}, \quad \text{for } \theta \in \Lambda_1 \tag{2.2}$$

$$P_F(\phi;\theta) \triangleq \int_\Gamma \phi(\mathrm{x}) p_\theta(\mathrm{x}) d\mathrm{x}, \quad \text{for } \theta \in \Lambda_0 \tag{2.3}$$

where Γ represents the observation space, and $\phi(\mathrm{x})$ denotes a generic decision rule (detector) that maps x into a real number in [0, 1], representing the probability of selecting $\mathcal{H}_1$ [22]. Then, the formulation of the restricted NP criterion is the following optimization problem [27]:

$$\max_\phi \int_{\Lambda_1} P_D(\phi;\theta) w_1(\theta) d\theta \tag{2.4}$$

$$\text{subject to } P_D(\phi;\theta) \geq \beta, \quad \forall\theta \in \Lambda_1 \tag{2.5}$$

$$P_F(\phi;\theta) \leq \alpha, \quad \forall\theta \in \Lambda_0 \tag{2.6}$$

where α is the false-alarm constraint, and β is the design parameter to compensate for the uncertainties in $w_1(\theta)$. In other words, a restricted NP decision rule is a rule that maximizes the average detection probability (obtained based on the prior distribution estimate $w_1(\theta)$) under the constraints on the worst-case detection and false-alarm probabilities. Worst-case detection probability should be higher than β, and false-alarm probability should be smaller than α.

Max-min and classical NP approaches are two special cases of the formulation in (2.4)-(2.6) [27]. In the case of full uncertainty in $w_1(\theta)$ (no prior information), the restricted NP problem reduces to the max-min problem and the following problem is considered [27]:

$$\begin{aligned} &\max_\phi \min_{\theta\in\Lambda_1} P_D(\phi;\theta) \\ &\text{subject to } P_F(\phi;\theta) \leq \alpha, \quad \forall\theta \in \Lambda_0 \end{aligned} \tag{2.7}$$

On the other hand, in the case of no uncertainty in $w_1(\theta)$ (perfect prior information), the restricted NP problem turns into the classical NP problem, which can be presented as follows [27]:

$$\max_{\phi} P_D^{avg}(\phi)$$
$$\text{subject to } P_F(\phi;\theta) \leq \alpha, \quad \forall\theta \in \Lambda_0 \tag{2.8}$$

where $P_D^{avg}(\phi) \triangleq \int_{\Lambda_1} P_D(\phi;\theta)w_1(\theta)d\theta$ defines the average detection probability.

According to the theoretical results in [34], [30], we can show that the optimal solution to (2.4)-(2.6) is in the form of an NP decision rule corresponding to the *least-favorable* distribution [27]. The least-favorable distribution can be obtained by combining the uncertain PDF $w_1(\theta)$ and any other PDF $\mu(\theta)$ as follows [27]:

$$v(\theta) = \lambda w_1(\theta) + (1-\lambda)\mu(\theta) \tag{2.9}$$

and the PDF $v(\theta)$ that corresponds to the minimum average detection probability can be found [27].

2.2.2 Characterization of Optimal Decision Rule

A detailed formulation of the restricted NP algorithm can be obtained by employing the definitions in (2) and (3) to reformulate the problem in ((2.4)-(2.6) as [27]

$$\max_{\phi} \int_{\Gamma} \phi(\mathrm{x})p_1(\mathrm{x})d\mathrm{x} \tag{2.10}$$
$$\text{subject to } \min_{\theta\in\Lambda_1} \int_{\Gamma} \phi(\mathrm{x})p_\theta(\mathrm{x})d\mathrm{x} \geq \beta \tag{2.11}$$
$$\max_{\theta\in\Lambda_0} \int_{\Gamma} \phi(\mathrm{x})p_\theta(\mathrm{x})d\mathrm{x} \leq \alpha \tag{2.12}$$

where $p_1(x) \triangleq \int_{\Lambda_1} p_\theta(x)w_1(\theta)d\theta$ is the PDF of the observation under $\mathcal{H}_1$, which is obtained based on the prior distribution estimate $w_1(\theta)$. The problem in (2.10)-(2.12) can be represented alternatively as follows [27]

$$\max_{\phi} \lambda \int_{\Gamma} \phi(\mathrm{x})p_1(\mathrm{x})d\mathrm{x} + (1-\lambda)\min_{\theta\in\Lambda_1} \int_{\Gamma} \phi(\mathrm{x})p_\theta(\mathrm{x})d\mathrm{x}$$
$$\text{subject to} \max_{\theta\in\Lambda_0} \int_{\Gamma} \phi(\mathrm{x})p_\theta(\mathrm{x})d\mathrm{x} \leq \alpha \tag{2.13}$$

where $0 \leq \lambda \leq 1$ is a design parameter that is selected according to β [27]. As a special case, if $\lambda = 0$ then the restricted NP reduces to max-min problem. Similarly, if $\lambda = 1$ then it reduces to classical NP problem [27].

If the parameter space for $\mathcal{H}_0$ is specified as $\Lambda_0 = 0$, then (2.13) becomes

$$\begin{aligned} &\max_{\phi} \lambda \int_{\Gamma} \phi(\mathrm{x}) p_1(\mathrm{x}) d\mathrm{x} + (1-\lambda) \min_{\theta \in \Lambda_1} \int_{\Gamma} \phi(\mathrm{x}) p_\theta(\mathrm{x}) d\mathrm{x} \\ &\text{subject to} \int_{\Gamma} \phi(\mathrm{x}) p_0(\mathrm{x}) d\mathrm{x} \leq \alpha \end{aligned} \tag{2.14}$$

Based on NP lemma [22], we can show that the solution of (2.14) is in the form of a likelihood ratio test (LRT) as follows [27]:

$$\phi^*(\mathrm{x}) = \begin{cases} 1, & \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta \geq \eta p_0(\mathrm{x}) \\ 0, & \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta < \eta p_0(\mathrm{x}) \end{cases} \tag{2.15}$$

where the threshold η is chosen such that the false-alarm rate is equal to α (i.e., $P_F(\phi^*) = \alpha$), $v(\theta) = \lambda w_1(\theta) + (1-\lambda)\mu(\theta)$, and $\mu(\theta)$ is to be obtained for the least-favorable distribution [27].

Proof ([27]): We can use the approach on page 24 of [22] to prove that $\int_\Gamma \phi^*(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} \geq \int_\Gamma \phi(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x}$ for any decision rule ϕ that satisfies $P_F(\phi) \leq \alpha$.

Let ϕ be any decision rule satisfying $P_F(\phi) \leq \alpha$ and let ϕ^* be any decision rule in the form of (2.15). From the definition of ϕ and ϕ^*, we have

$$(\phi^*(\mathrm{x}) - \phi(\mathrm{x})) \left(\int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta - \eta p_0(\mathrm{x}) \right) \geq 0, \forall \mathrm{x} \tag{2.16}$$

Hence,

$$\int_\Gamma (\phi^*(\mathrm{x}) - \phi(\mathrm{x})) \left(\int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta - \eta p_0(\mathrm{x}) \right) d\mathrm{x}) \geq 0 \tag{2.17}$$

By rearranging the terms in (2.17) we obtain,

$$\begin{aligned} &\int_\Gamma \phi^*(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} - \int_\Gamma \phi(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} \\ &\geq \eta [\underbrace{\int_\Gamma \phi^*(\mathrm{x}) p_0(\mathrm{x})) dx}_{P_F(\phi)^* = \alpha} - \underbrace{\int_\Gamma \phi(\mathrm{x}) p_0(\mathrm{x})) dx}_{P_F(\phi) \leq \alpha}] \end{aligned} \tag{2.18}$$

Then,

$$\int_{\Gamma} \phi^*(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} - \int_{\Gamma} \phi(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} \geq 0 \tag{2.19}$$

$$\int_{\Gamma} \phi^*(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} \geq \int_{\Gamma} \phi(\mathrm{x}) \int_{\Lambda_1} p_\theta(\mathrm{x}) v(\theta) d\theta d\mathrm{x} \tag{2.20}$$

which completes the proof.

2.2.3 Algorithm

The following algorithm is proposed to obtain the optimal restricted NP decision rule in [27]:

1) Obtain $P_D(\phi^*_{\theta_1}; \theta)$ for all $\theta_1 \in \Lambda_1$, where $\phi^*_{\theta_1}$ denotes the α-level NP decision rule corresponding to $v(\theta) = \lambda w_1(\theta) + (1-\lambda)\delta(\theta - \theta_1)$ as in (2.15).

2) Calculate

$$\theta_1^* = \arg \min_{\theta_1 \in \Lambda_1} f(\theta_1) \tag{2.21}$$

where

$$f(\theta_1) \triangleq \lambda \int_{\Lambda_1} w_1(\theta) P_D(\phi^*_{\theta_1}; \theta) d\theta + (1-\lambda) P_D(\phi^*_{\theta_1}; \theta_1). \tag{2.22}$$

3) If $P_D(\phi^*_{\theta_1^*}; \theta_1^*) = \min_{\theta \in \Lambda_1} P_D(\phi^*_{\theta_1^*}; \theta)$, output $\phi^*_{\theta_1^*}$ as the solution of the restricted NP problem; otherwise, the solution does not exist.

It is important to note that $f(\theta_1)$ in (2.22) is the average detection probability corresponding to $v(\theta) = \lambda w_1(\theta) + (1-\lambda)\delta(\theta - \theta_1)$ [27].

We can explain the above algorithm in detail as follows: First a new value for θ_1 is selected from the parameter set Λ_1 corresponding to the presence of primary user. Then, PDF $v(\theta) = \lambda w_1(\theta) + (1-\lambda)\delta(\theta - \theta_1)$ is constructed by enhancing the uncertain prior distribution $w_1(\theta)$ with the selected parameter θ_1. After that, the α-level NP decision rule $\phi^*_{\theta_1}$ corresponding to $v(\theta)$ is obtained as shown in equation (2.15). By utilizing equation 2.2 the detection probability

$P_D(\phi^*_{\theta_1};\theta)$ corresponding to decision rule $\phi^*_{\theta_1}$ is computed as a function of θ and, then the average detection probability $f(\theta_1)$ is computed as a function of θ_1 by integrating $P_D(\phi^*_{\theta_1};\theta)$ over the enhanced PDF $v(\theta)$ as shown in (2.22). This procedure continues until finding $\theta^*_1 \in \Lambda_1$ which minimizes $f(\theta_1)$ (i.e., $\theta^*_1 = \arg\min_{\theta_1\in\Lambda_1} f(\theta_1)$. If $\theta^*_1 = \arg\min_{\theta\in\Lambda_1} P_D(\phi^*_{\theta^*_1};\theta)$, then the algorithm stops and $\phi^*_{\theta^*_1}$ is outputted as the solution of the restricted NP problem; otherwise, the solution does not exist.

The main complexity of the above algorithm is due to the calculation the of $f(\theta_1)$, which can be restated as follows:

$$f(\theta_1) = \lambda \int_{-\infty}^{\infty} \phi^*_{\theta_1}(\mathrm{x}) \int_{\Lambda_1} w_1(\theta) p_\theta(\mathrm{x}) d\theta d\mathrm{x} + (1-\lambda) \int_{-\infty}^{\infty} \phi^*_{\theta_1}(\mathrm{x}) p_{\theta_1}(\mathrm{x}) d\mathrm{x}. \quad (2.23)$$

$\int_{\Lambda_1} w_1(\theta) p_\theta(\mathrm{x}) d\theta$ term is the main reason of complexity in calculation of $f(\theta_1)$, so we can reduce the complexity by simplifying this term.

Chapter 3

Application of the Restricted Neyman-Pearson Approach to Spectrum Sensing in Cognitive Radio

In this section, the application of the restricted NP approach to the spectrum sensing problem in cognitive radio systems is discussed. Spectrum sensing in the presence of Gaussian noise and non-Gaussian noise is presented. In addition to the restricted NP approach, GLRT and energy detection (ED) approaches for our problem are provided so that their performance can be compared with that of the restricted NP approach.

3.1 Spectrum Sensing in the Presence of Gaussian Noise

If the transmission policies of the primary users are not known, energy-detection methods are considered to be appropriate for channel sensing. In this case, the spectrum sensing problem can be formulated as a hypothesis testing problem between the noise n_i and the signal s_i over Gaussian channels [40],

$$\begin{aligned} \mathcal{H}_0 &: x_i = n_i, \quad i = 1, 2, ..., N, \\ \mathcal{H}_1 &: x_i = s_i + n_i, \quad i = 1, 2, ..., N. \end{aligned} \tag{3.1}$$

where s_i is the sum of PS users faded signals received by the secondary user (SU), and n_i's are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian noise samples with zero mean and variance $E\{|n_i|^2\} = \sigma_n^2$. We also assume that s_i has circularly symmetric distribution with zero-mean and variance σ_s^2. Employing energy detection, the detector for the above hypothesis testing problem is given by

$$X = \frac{1}{N}\sum_{i=1}^{N} |x_i|^2 \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \lambda \tag{3.2}$$

where λ is the detection threshold. For a sufficiently large value of N, X can be approximated as a Gaussian random variable by invoking Central Limit Theorem and the mean and variance of X under $\mathcal{H}_0$ and $\mathcal{H}_1$ are given as follows (Please see Appendix A for the proof):

$$E\{X\} = \begin{cases} \sigma_n^2, & \mathcal{H}_0 \\ \sigma_s^2 + \sigma_n^2, & \mathcal{H}_1 \end{cases} \tag{3.3}$$

$$Var\{X\} = \begin{cases} \sigma_n^4/N, & \mathcal{H}_0 \\ (E\{|s|^4\} + 2\sigma_n^4 - (\sigma_s^2 - \sigma_n^2)^2)/N, & \mathcal{H}_1 \end{cases} \tag{3.4}$$

By using the expressions above, the probability distributions under $\mathcal{H}_0$ and $\mathcal{H}_1$ can be written as

$$\begin{aligned} &\mathcal{H}_0 : X \sim \mathcal{N}(\sigma_n^2, \sigma_n^4/N) \\ &\mathcal{H}_1 : X \sim \mathcal{N}(\sigma_s^2 + \sigma_n^2, (E\{|s|^4\} + 2\sigma_n^4 - (\sigma_s^2 - \sigma_n^2)^2)/N) \end{aligned} \tag{3.5}$$

3.1.1 Restricted NP Approach

In cognitive radio networks, the signals from PS users are usually unknown. Hence, the unknown parameter $\boldsymbol{\theta}$ in the restricted NP algorithm can be defined as

$$\boldsymbol{\theta} = [\sigma_s^2 \quad E\left\{|s|^4\right\}] \tag{3.6}$$

Above, we do not assume any special distribution for s_i. We only assume that it has circularly symmetric distribution with zero-mean and variance σ_s^2. In addition to general scenarios, a special case in which s_i has a complex Gaussian distribution can be considered. In this case, $E\left\{|s|^4\right\}] = 2\sigma_s^4$; so the unknown parameter θ is simplified and it becomes $\theta = \sigma_s^2$. The Gaussian assumption for s_i can be justified in some cases in practice. For example, the number of active primary signals can be very large and in such a case we can assume s_i to be Gaussian, because it is the sum of a large number of faded signals [40].

In practice, some information about the prior distribution of $\boldsymbol{\theta}$ is available in many cases. However, that information is not perfect most of the time, which means that prior distribution of $\boldsymbol{\theta}$ is uncertain. Due to this uncertainty in the prior distribution of the parameter under $\mathcal{H}_1$, the restricted NP approach is very suitable in the solution of this problem. Using the restricted NP approach, imperfect prior information is utilized. Additionally, certain level of detection probability is guaranteed in order to limit the interference to primary users.

Using (3.5), we can express $p_{\boldsymbol{\theta}}(\mathrm{x})$ as follows:

$$p_{\boldsymbol{\theta}}(\mathrm{x}) = \frac{\exp\left\{-\frac{(\mathrm{x}-\sigma_s^2-\sigma_n^2)^2}{2(E\{|s|^4\}+2\sigma_n^4-(\sigma_s^2-\sigma_n^2)^2)/N}\right\}}{\sqrt{2\pi(E\left\{|s|^4\right\}+2\sigma_n^4-(\sigma_s^2-\sigma_n^2)^2)/N}} \tag{3.7}$$

where $\boldsymbol{\theta} = [\sigma_s^2 \quad E\left\{|s|^4\right\}]$.

In order to evaluate $f(\boldsymbol{\theta}_1)$ in (2.23), we need to calculate $\int_{\Lambda_1} w_1(\boldsymbol{\theta}) p_{\boldsymbol{\theta}}(\mathrm{x}) d\boldsymbol{\theta}$. Hence, $w_1(\boldsymbol{\theta})$ should be known. However, in practice the prior distribution of $\boldsymbol{\theta}$ cannot be known perfectly, instead some imperfect prior distribution can be available based on previous measurements and/or geographical information. We can consider different prior distributions for various scenarios. Calculation complexity of $f(\boldsymbol{\theta}_1)$ depends on the structure of the assumed prior distribution.

If we assume the PS users' signals are Gaussian, then $p_\theta(\mathrm{x})$ in (3.7) includes only a single unknown parameter, which is $\theta = \sigma_s^2$. In this case, $p_\theta(\mathrm{x})$ is reduces to the following expression:

$$p_\theta(\mathrm{x}) = \frac{\exp\left\{-\frac{(x-\sigma_s^2-\sigma_n^2)^2}{2(\sigma_s^2+\sigma_n^2)^2/N}\right\}}{\sqrt{2\pi(\sigma_s^2+\sigma_n^2)^2/N}} \tag{3.8}$$

In the Gaussian PS users case, the numerical evaluation of the integral $\int_{\Lambda_1} w_1(\theta)p_\theta(\mathrm{x})d\theta$ is not very difficult for any prior distribution. For the Gaussian case, we can consider various prior probability distributions for $\theta = \sigma_s^2$ such as uniform, Rayleigh and exponential. On the other hand, $\int_{\Lambda_1} w_1(\boldsymbol{\theta})p_{\boldsymbol{\theta}}(\mathrm{x})d\boldsymbol{\theta}$ can be difficult to calculate for the generic non-Gaussian case because it is in fact a double integral. The easiest probability distribution that can be considered is uniform distribution. When different distributions are considered other than the uniform distribution, the expression can become complex. If we assume uniform distribution for $\boldsymbol{\theta}$ over $[a_1, a_2] \times [b_1, b_2]$, then the integral becomes

$$\int_{\Lambda_1} w_1(\boldsymbol{\theta})p_{\boldsymbol{\theta}}(\mathrm{x})d\boldsymbol{\theta} = \frac{1}{(a_2-a_1)(b_2-b_1)} \int_{b_1}^{b_2}\int_{a_1}^{a_2} \frac{\exp\left\{-\frac{(\mathrm{x}-a-\sigma_n^2)^2}{2(b+2\sigma_n^4-(a-\sigma_n^2)^2)/N}\right\}}{\sqrt{2\pi(b+2\sigma_n^4-(a-\sigma_n^2)^2)/N}} da\, db \tag{3.9}$$

3.1.2 GLRT Approach

In this section, a brief description of the generalized likelihood ratio test (GLRT) approach is provided in order to compare performance of the proposed restricted NP based spectrum algorithm with that of the GLRT based spectrum sensing.

GLRT is one of the useful methods that can be used in composite hypothesis-testing problems. In GLRT, first maximum likelihood estimates (MLEs) of the unknown parameters under $\mathcal{H}_0$ and $\mathcal{H}_1$ are found, then the GLRT statistic is formed [41]. This procedure can be formulated as follows:

$$L(\mathrm{x}) = \frac{\max_{\theta\in\Lambda_1} p_\theta(\mathrm{x})}{\max_{\theta\in\Lambda_0} p_\theta(\mathrm{x})} \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \lambda \tag{3.10}$$

where the threshold λ is chosen such that the probability of false alarm satisfies $P(\mathcal{H}_1|\mathcal{H}_0) = P(L(\mathrm{x}) \geq \lambda|\mathcal{H}_0) = \alpha$.

If we return to our original binary hypothesis-testing problem in (3.5), the GLRT for it can be written using (3.7) as follows:

$$\frac{\max_{\boldsymbol{\theta}} \frac{\exp\left\{-\frac{(\mathrm{x}-\sigma_s^2-\sigma_n^2)^2}{2(E\{|s|^4\}+2\sigma_n^4-(\sigma_s^2-\sigma_n^2)^2)/N}\right\}}{\sqrt{2\pi(E\{|s|^4\}+2\sigma_n^4-(\sigma_s^2-\sigma_n^2)^2)/N}}}{\frac{\exp\left\{-\frac{(\mathrm{x}-\sigma_n^2)^2}{2\sigma_n^4/N}\right\}}{\sqrt{2\pi\sigma_n^4/N}}} \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \eta_g \tag{3.11}$$

where $\boldsymbol{\theta} = [\sigma_s^2 \quad E\{|s|^4\}]$. Note that since the parameter space for $\mathcal{H}_0$ is specified as $\Lambda_0 = 0$, no maximization operator is needed in the denominator of GLRT. We can rearrange (3.11) to get more compact expression

$$\max_{\boldsymbol{\theta}} \frac{\exp\left\{\frac{(\mathrm{x}-\sigma_n^2)^2}{2\sigma_n^4/N} - \frac{(\mathrm{x}-\sigma_s^2-\sigma_n^2)^2}{2(E\{|s|^4\}+2\sigma_n^4-(\sigma_s^2-\sigma_n^2)^2)/N}\right\}}{\sqrt{(E\{|s|^4\}+2\sigma_n^4-(\sigma_s^2-\sigma_n^2)^2}} \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \widetilde{\eta}_g \tag{3.12}$$

where $\widetilde{\eta}_g \triangleq \eta_g/\sigma_n^2$ and it should be chosen such that the false alarm probability is equal to α.

3.1.3 Energy Detection Approach

In addition to GLRT, we can also consider the energy detection algorithm, which has a low complexity, to compare performance of it with that of the restricted NP approach. In this alternative algorithm, observation X ($X = \frac{1}{N}\sum_{i=1}^{N}|x_i|^2$), whose distribution is given in (3.5), is directly compared to a threshold η_e. The threshold η_e is chosen such that the false alarm probability is equal to α, that is,

$$P(X > \eta_e|\mathcal{H}_0) = \int_{\eta_e}^{\infty} p_0(\mathrm{x})\, d\mathrm{x} = \alpha \tag{3.13}$$

where $p_0(\mathrm{x}) = \frac{1}{\sqrt{2\pi\sigma_n^4/N}}\exp\left\{-\frac{(\mathrm{x}-\sigma_n^2)^2}{2\sigma_n^4/N}\right\}$. After inserting $p_0(\mathrm{x})$ expression in (3.13) and performing some manipulations, we obtain the energy detection threshold

η_e as

$$\eta_e = \frac{\sigma_n^2}{\sqrt{N}} Q^{-1}(\alpha) + \sigma_n^2 \tag{3.14}$$

where Q^{-1} denotes the inverse of the Q-function, which is defined as $Q(x) = (1/\sqrt{2\pi}) \int\limits_x^\infty e^{-t^2/2}\, dt$. Then, the detection probability for the energy detection approach can be calculated as follows:

$$P(X > \eta_e | \mathcal{H}_1) = \int\limits_{\eta_e}^{\infty} p_{\boldsymbol{\theta}}(\mathrm{x})\, d\mathrm{x} = Q\left(\frac{\eta_e - \sigma_s^2 - \sigma_n^2}{\sqrt{(E\{|s|^4\} + 2\sigma_n^4 - (\sigma_s^2 - \sigma_n^2)^2)/N}}\right) \tag{3.15}$$

3.2 Spectrum Sensing over Non-Gaussian Channels

3.2.1 Single Observation Case

As mentioned in the previous chapter, the most popular techniques for spectrum sensing are matched filtering, energy detection, and cyclostationary feature detection. These schemes differ in the required amount of a priori knowledge about the primary user signal, but they are usually all optimized under the assumption that the primary user signal is only impaired by additive white Gaussian noise (AWGN). Although the noise distribution is often assumed to be Gaussian which is more tractable mathematically, in practice we cannot always model noise as Gaussian. In other words, the received signal at the cognitive radios may also be impaired by non-Gaussian noise. Non-Gaussian noise impairments may include man-made impulsive noise, co-channel interference from other cognitive radios, and interference from ultra-wideband systems. Spectrum sensing for cognitive radio networks in the presence of non-Gaussian noise has been studied by several researchers recently such as [42], [43], [44].

Let us consider the following hypothesis-testing problem:

$$\begin{aligned}\mathcal{H}_0 &: X = N \\ \mathcal{H}_1 &: X = \theta + N\end{aligned} \tag{3.16}$$

where θ represents the unknown parameter, and N is the noise. Similar to the Gaussian noise case, the prior distribution of θ under $\mathcal{H}_1$, which is denoted by $w_1(\theta)$, may not be perfect and can include certain errors (uncertainty). The noise N is modeled as non-Gaussian noise. In particular, a generic Gaussian mixture noise model is considered in this thesis. This type of noise is usually derived from man-made impulsive noise interference. A Gaussian mixture noise model is a weighted sum of N_m component Gaussian noises as given by the following PDF:

$$p_N(n) = \sum_{i=1}^{N_m} \frac{\nu_i}{\sqrt{2\pi}\epsilon_i} \exp\left(-\frac{(n-\mu_i)^2}{2\epsilon_i^2}\right) \tag{3.17}$$

where N_m is the number of components, and μ_i, ϵ_i^2, ν_i are the mean, variance, and weight of each component, respectively.

In practice, some prior information about probability distribution of θ in (3.16) is available to secondary users. This prior knowledge is usually obtained by using previous measurements and/or by utilizing pilot signals. However, that prior information can include uncertainties. Hence, we can adopt the restricted NP approach for this problem due to this uncertainty in the prior PDF of θ, which is denoted by $w_1(\theta)$.

Similar to the Gaussian case, the performance of the restricted NP approach is compared against the GLRT and ED approaches. Simulations for all approaches are presented in the numerical results section. As mentioned in the previous section, the GLRT expression can be obtained based on (3.16) and (3.17) as follows:

$$\max_{\theta} \frac{\sum_{i=1}^{N_m} \nu_i \exp(-\frac{(n-\theta-\mu_i)^2}{2\epsilon_i^2})}{\sum_{i=1}^{N_m} \nu_i \exp(-\frac{(n-\mu_i)^2}{2\epsilon_i^2})} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \eta_g \tag{3.18}$$

where η_g is chosen such that the false alarm probability is equal to α.

3.2.2 Multiple Observations Case

In the previous section, we assumed that there is only a single observation available to the secondary users.In this section we consider the case of multiple observations. The model in (3.16) can be extended to multiple observations case as follows:

$$\mathcal{H}_0 : X_i = N_i, \quad i = 1, 2, ..., M$$
$$\mathcal{H}_1 : X_i = \theta + N_i, \quad i = 1, 2, ..., M \tag{3.19}$$

where θ is the unknown parameter, M is the number of observations, and N_i's are independent and identically distributed according to the generic Gaussian mixture PDF in (3.17). Similar to the single observation case, the prior distribution of θ, denoted by $w_1(\theta)$, is not perfect and includes some uncertainties. Therefore, the restricted NP approach can be employed also for this case.

As in the previous case, the performance of the restricted NP is compared to that of GLRT and energy detection approaches. The GLRT expression for this case can be expressed as follows:

$$\max_{\theta} \frac{\prod_{j=1}^{M} \sum_{i=1}^{N_m} \frac{\nu_i}{\sqrt{2\pi}\epsilon_i} \exp(-\frac{(x_j-\theta-\mu_i)^2}{2\epsilon_i^2})}{\prod_{j=1}^{M} \sum_{i=1}^{N_m} \frac{\nu_i}{\sqrt{2\pi}\epsilon_i} \exp(-\frac{(x_j-\mu_i)^2}{2\epsilon_i^2})} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \eta_g \tag{3.20}$$

where η_g is chosen such that the false alarm probability is equal to α.

On the other hand, energy detector's task is to compare the total energy of the observations to a threshold; i.e, $\sum_{i=1}^{M} X_i^2 \geq \eta_e$. The threshold η_e is chosen such that the false alarm probability is equal to α as follows:

$$P\left\{\sum_{i=1}^{M} X_i^2 > \eta_e | \mathcal{H}_0\right\} = \alpha \tag{3.21}$$

Chapter 4

NUMERICAL RESULTS

In the previous chapters, we have introduced the problem of spectrum sensing in cognitive radio systems, presented an analysis of the problem with existing approaches and proposed a novel algorithm based on the restricted NP approach. The proposed spectrum sensing algorithm is expected to improve performance both in Gaussian and non-Gaussian noise channels. Therefore, in this chapter we present numerical and simulation results in order to show the accuracy and the robustness of the proposed restricted NP algorithm for various scenarios. In addition to the simulations of the restricted NP approach, simulations for two existing approaches; namely, GLRT and energy detection, are also performed in order to provide comparisons.

This chapter is split into two main sections. In the first one, simulation results for these three spectrum sensing methods are obtained in the presence of Gaussian noise. To do this, first we consider Gaussian primary user signals and simulation results for this single unknown parameter case are presented. Then, we assume no special distributions for PS users' signals and generic two-parameter case simulations are provided. In the second section, non-Gaussian noise channel is considered. In this part, first we provide simulations for the single observation case. Then, we look at how the performances of the detectors are changing when multiple observations are available at the secondary users.

4.1 Simulation Results for Spectrum Sensing in the Presence of Gaussian Noise

In this section, Gaussian primary signals are considered, and simulation results for the single unknown parameter case $\theta = \sigma_s^2$ are presented. The unknown parameter θ has a parameter space $\Lambda = [a, b]$. It is assumed to be modeled as a random variable with a PDF in the form of uniform distribution over the interval $[a, b]$; that is, $w_1(\theta) = 1/(b-a)$ for $\theta \in [a, b]$. Using this prior PDF, $v(\theta)$ in the restricted NP algorithm can be expressed as

$$v(\theta) = \frac{\lambda}{b-a} + (1-\lambda)\delta(\theta - \theta_1), \quad \theta \in \Lambda_1 \tag{4.1}$$

$f(\theta_1)$ in (2.22) can be calculated, using $P_D(\phi^*_{\theta_1}; \theta) = \int \phi^*_{\theta_1}(\mathrm{x}) p_\theta(\mathrm{x}) d\mathrm{x}$, as

$$\begin{aligned} f(\theta_1) = {} & \frac{\lambda}{b-a} \int_{\Lambda_1} \int_{-\infty}^{\infty} \phi^*_{\theta_1}(\mathrm{x}) \frac{\exp\left\{-\frac{(x-\sigma_s^2-\sigma_n^2)^2}{2(\sigma_s^2+\sigma_n^2)^2/N}\right\}}{\sqrt{2\pi(\sigma_s^2+\sigma_n^2)^2/N}} d\mathrm{x} d\theta \\ & + (1-\lambda) \int_{-\infty}^{\infty} \phi^*_{\theta_1}(\mathrm{x}) \frac{\exp\left\{-\frac{(x-\theta_1-\sigma_n^2)^2}{2(\theta_1+\sigma_n^2)^2/N}\right\}}{\sqrt{2\pi(\theta_1+\sigma_n^2)^2/N}} d\mathrm{x} \end{aligned} \tag{4.2}$$

Based on (4.2), we can solve the optimization problem in the second step of the restricted NP algorithm, and in the third step the resulting minimizer is used to check whether the minimum value of $P_D(\phi^*_{\theta_1^*}; \theta)$ over $\theta \in [a, b]$ is equal to $P_D(\phi^*_{\theta_1^*}; \theta_1^*)$ or not. If they are equal, $\phi^*_{\theta_1^*}$ is output as the solution of the algorithm. Otherwise, there is no solution for this θ_1 and algorithm proceeds with the new θ_1 value.

In the simulations, $a = 0.5$ and $b = 1$ are used. Also, the other parameters are set to $\sigma_n^2 = 0.5$, $N = 1$ and $\alpha = 0.1$. Figure 4.1 shows the detection probability versus the parameter value θ, and Figure 4.2 plots the worst-case (minimum)

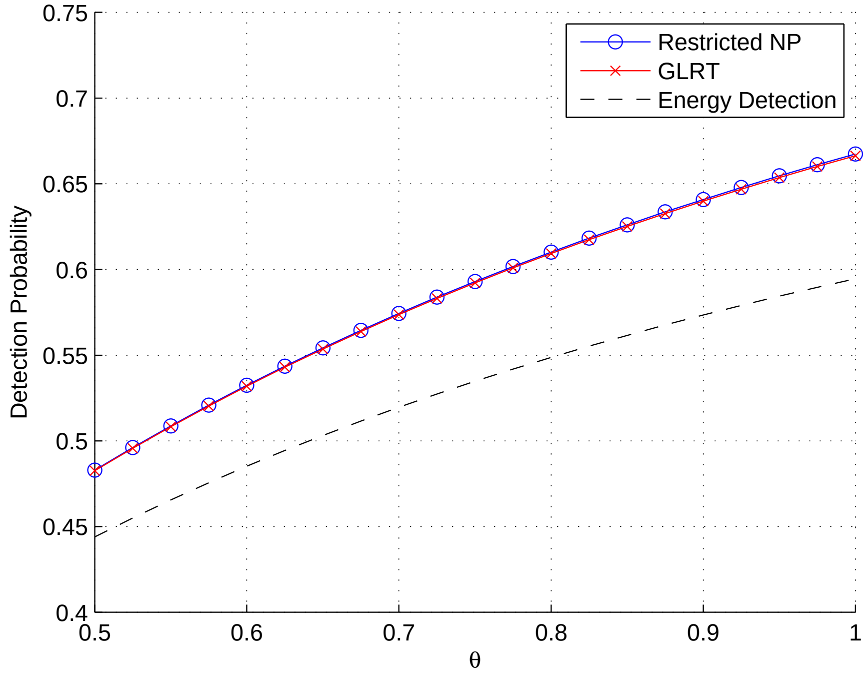


Figure 4.1: Detection probability versus θ for the three approaches for $a = 0.5$, $b = 1$, $\sigma_n^2 = 0.5$, $N = 1$ and $\alpha = 0.1$.

and average detection probabilities versus λ for the three approaches. As can be seen from Figure 4.2 the performance of the restricted NP approach stays the same for all λ values in this scenario, so the results in Figure 4.1 are valid for all the restricted NP solutions in this case. Both Figure 4.1 and Figure 4.2 shows that the GLRT and the restricted NP approaches have the same performance for all parameter values, and their performance are significantly much better than the performance of the energy detection approach. From these results we observe that the same decision rule is obtained for all values of λ for this problem. That is, changing λ value does not change the performance of the restricted NP approach for this problem.

As another example, the previous scenario is considered for $N = 5$. Figure 4.3 indicates the probability of detection versus θ for this case and in Figure 4.4 the worst-case (minimum) and average detection probabilities versus λ are plotted. In general, the same observations as in the previous example are made.

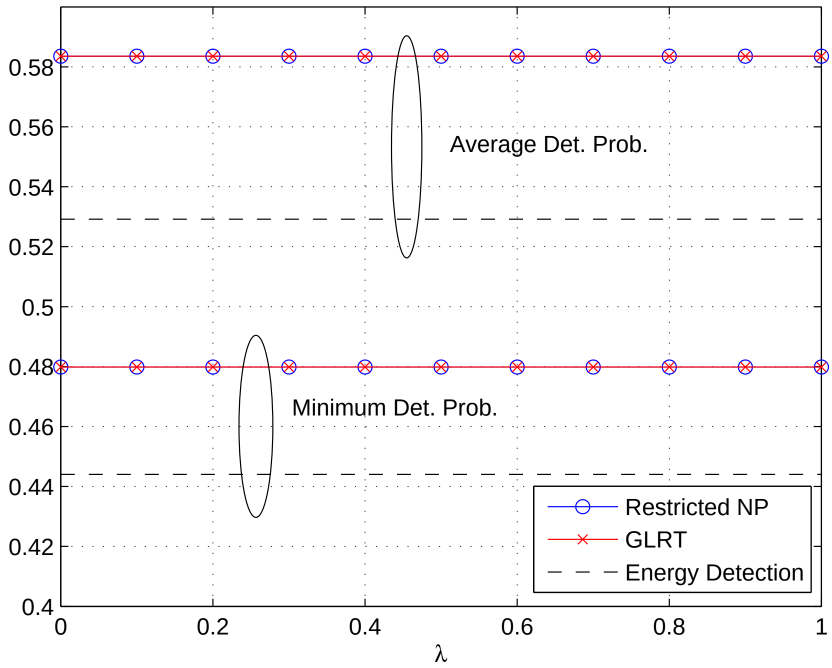


Figure 4.2: Detection probability versus λ for the three approaches for $a = 0.5$, $b = 1$, $\sigma_n^2 = 0.5$, $N = 1$ and $\alpha = 0.1$.

However, detection probabilities increase significantly due to the increase in the number of samples. Also, differences between the performance of the energy detection and other approaches decrease remarkably as the number of samples increase. Actually, they almost have the same performance if we further increase the number of samples (see Figure 4.5).

In addition to the one parameter case, simulations are performed also for the generic two-parameter case where the parameters are $[\theta_{11}\ \theta_{12}] = [\sigma_s^2 \quad E\{|s|^4\}]$. As in the one parameter case, the prior distributions of parameters θ_{11} and θ_{12} are assumed uniform over $[a, b]$ and $[c, d]$, respectively. In the simulations, $a = b = c = d = 0.5$ are used; that is, the parameter set is defined as $\Lambda_1 = [0.5, 1] \times [0.5, 1]$. Also, the other parameters are set to $\sigma_n^2 = 0.5$, $N = 1$, and $\alpha = 0.2$. Figure 4.6 shows the detection probabilities versus the parameters for the restricted NP, GLRT, and energy detection approaches. Similar to the one parameter case, the restricted NP and the GLRT techniques achieve a larger average and minimum

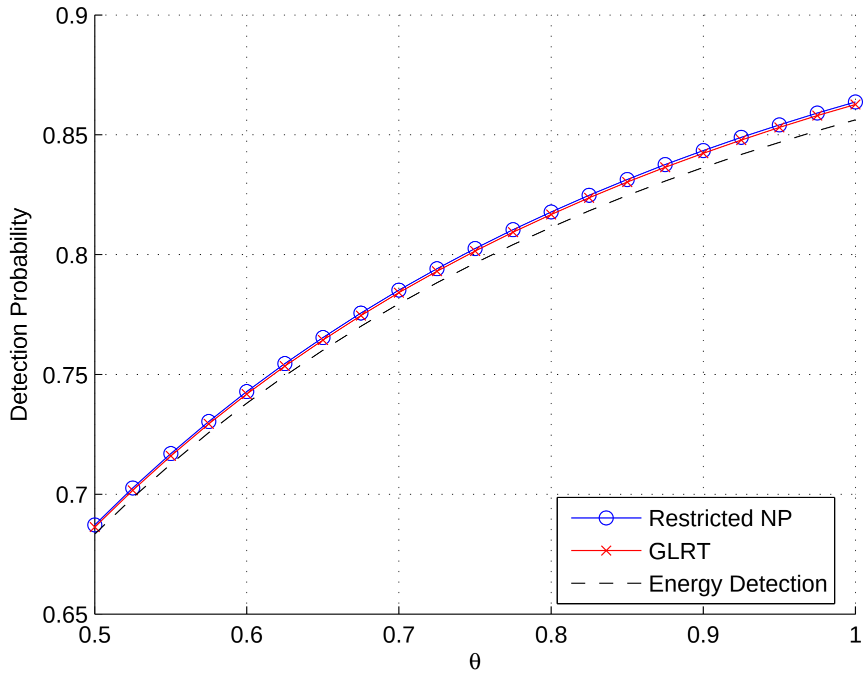


Figure 4.3: Detection probability versus θ for the three approaches for $a = 0.5$, $b = 1$, $\sigma_n^2 = 0.5$, $N = 5$ and $\alpha = 0.1$.

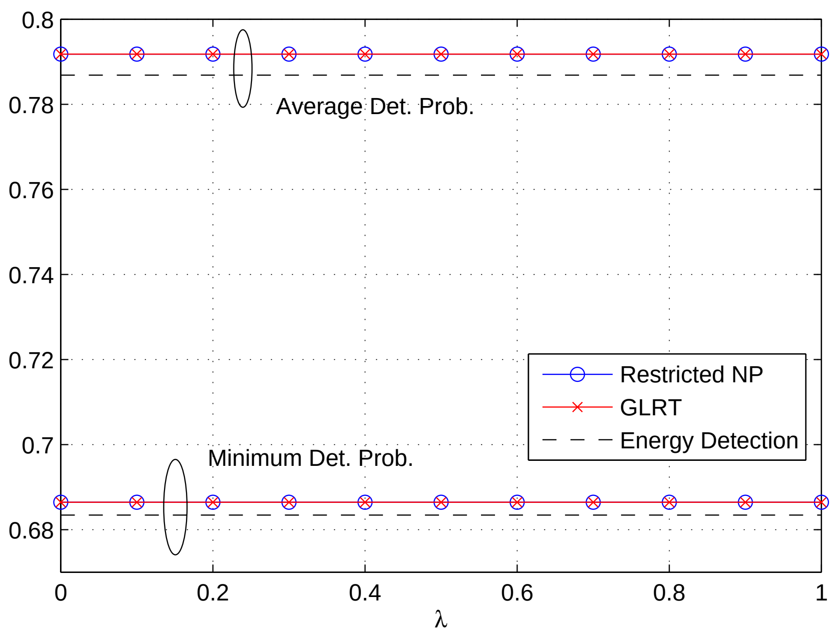


Figure 4.4: Detection probability versus λ for the three approaches for $a = 0.5$, $b = 1$, $\sigma_n^2 = 0.5$, $N = 5$ and $\alpha = 0.1$.

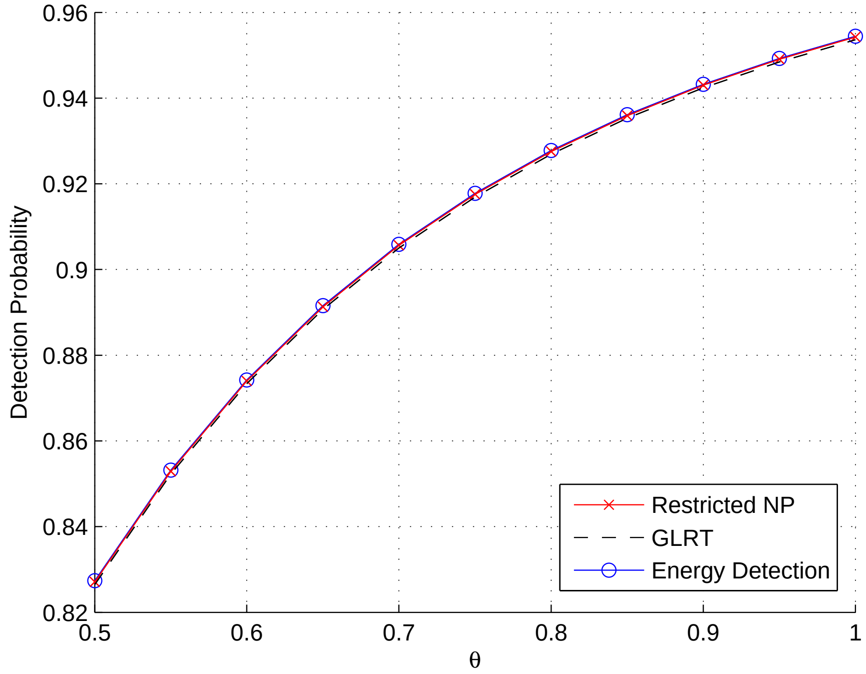


Figure 4.5: Detection probability versus θ for the three approaches for $a = 0.5$, $b = 1$, $\sigma_n^2 = 0.5$, $N = 10$ and $\alpha = 0.1$.

detection probabilities than the energy detection approach and they perform almost similarly at all parameter values.

In conclusion, for the spectrum sensing problem specified in (3.5), the restricted NP approach does not provide any significant improvements over the existing GLRT approach. Hence, we should consider different scenarios in which the restricted NP approach can provide unique advantages. This is performed in the next section.

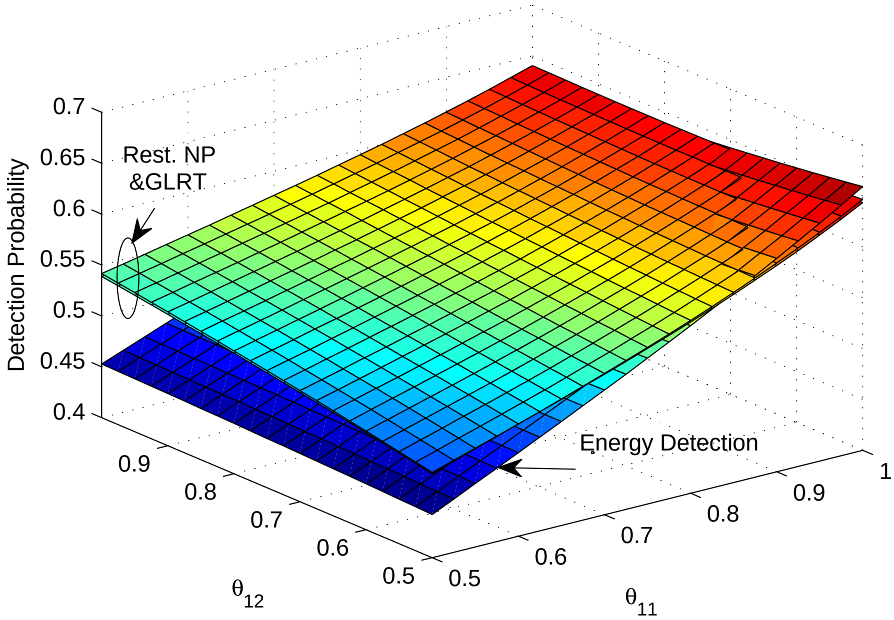


Figure 4.6: Detection probability versus θ_{11} and θ_{12} for the three approaches for $\sigma_n^2 = 0.5$, $N = 1$ and $\alpha = 0.2$. The restricted NP and the GLRT approaches have almost the same performance, while the energy detection results in lower detection probability for most parameter values.

4.2 Simulation Results for Spectrum Sensing over Non-Gaussian Channels

4.2.1 Single Observation Case

In this section, we consider the Gaussian mixture noise models and present simulation results for this scenario. For the problem formulation (3.16) in Chapter 3, θ is modeled as a random variable with a PDF in the form of uniform distribution over the interval $[a, b]$; that is, the parameter set under $\mathcal{H}_1$ is $\Lambda_1 = [a, b]$. Since noise has Gaussian mixture distribution, the conditional PDF of X for a given value of θ can be expressed as

$$p_\theta(\mathrm{x}) = \sum_{i=1}^{N_m} \frac{\nu_i}{\sqrt{2\pi}} \exp\left(-\frac{(\mathrm{x} - \theta - \mu_i)^2}{2\epsilon_i^2}\right) \tag{4.3}$$

In the simulations, three different Gaussian mixture noises are considered, and their parameters are provided in Table 4.1.

# of Gaussian components (N_m)	Mean values (μ_i's)	Weights (ν_i's)	Standard Deviations (ϵ_i's)
4	[-0.9 -0.5 0.5 0.9]	[0.3 0.2 0.2 0.3]	[0.1 0.1 0.1 0.1]
3	[-1 0 1]	[0.25 0.5 0.25]	[0.2 0.2 0.2]
2	[0 0]	[0.5 0.5]	[0.5 2]

Table 4.1: Gaussian mixture noise parameters.

In the simulations, $a = 1$ and $b = 2$ are used as the interval boundaries of θ. Also, the false alarm constraint is set to $\alpha = 0.1$. In Fig. 4.7 Gaussian mixture noise with parameters $N_m = 4$, $\nu_1 = \nu_4 = 0.3$, $\nu_2 = \nu_3 = 0.2$, $\mu_1 = -\mu_4 = 0.9$, $\mu_2 = -\mu_3 = 0.5$, $\epsilon_i = 0.1 \,\forall\, i$ is considered, and detection probabilities are plotted versus θ for the GLRT and the restricted NP approaches with various λ values. As a result of simulations, we observe that the minimum value of the detection probability; i.e., the worst-case detection probability, occurs around $\theta = 1.4$ for all the algorithms. As seen from Figure 4.7, the detection probability of the restricted NP for $\lambda = 0.2$ has the highest worst-case detection probability, and as λ increases the worst-case detection probability decreases. Hence, if a larger minimum detection probability is necessary, the restricted NP approach with a small value of λ should be employed. Conversely, if larger average detection probability is our concern, then λ should be increased because the average detection probability increases with increasing λ as seen from Figure 4.8. Figure 4.8 also indicates the trade-off between the worst-case detection probability and the average detection probability in detail. As λ goes from 0.2 to 1, the average detection probability increases and the minimum detection probability reduces. If we analyze the figure, we can infer that $\lambda = 0.5$ can be a reasonable choice in terms of both minimum and average detection probabilities because the average detection probability is as high as that of the GLRT, while the minimum detection probability is significantly larger than that of the GLRT.

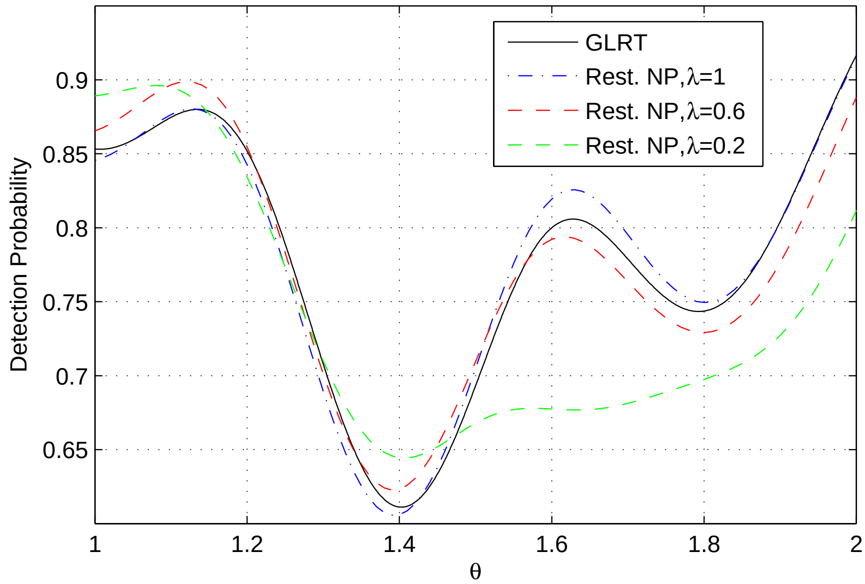


Figure 4.7: Detection probability versus θ for the restricted NP and GLRT approaches, where the parameters of the Gaussian mixture noise are set to $N_m = 4$, $\nu_1 = \nu_4 = 0.3$, $\nu_2 = \nu_3 = 0.2$, $\mu_1 = -\mu_4 = 0.9$, $\mu_2 = -\mu_3 = 0.5$, and $\epsilon_i = 0.1\ \forall\ i$.

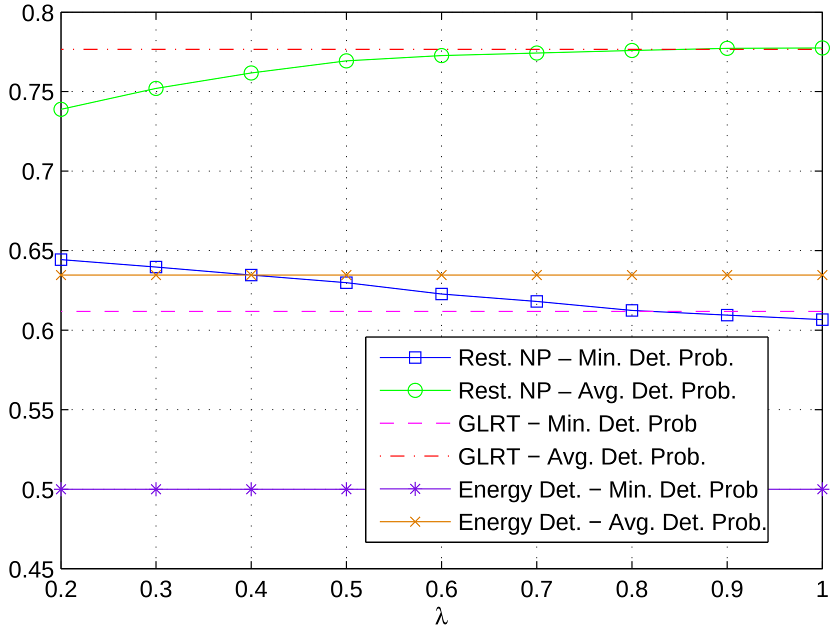


Figure 4.8: Detection probability versus λ for the restricted NP, GLRT and ED approaches, where the parameters of the Gaussian mixture noise are set to $N_m = 4$, $\nu_1 = \nu_4 = 0.3$, $\nu_2 = \nu_3 = 0.2$, $\mu_1 = -\mu_4 = 0.9$, $\mu_2 = -\mu_3 = 0.5$, and $\epsilon_i = 0.1\ \forall\ i$.

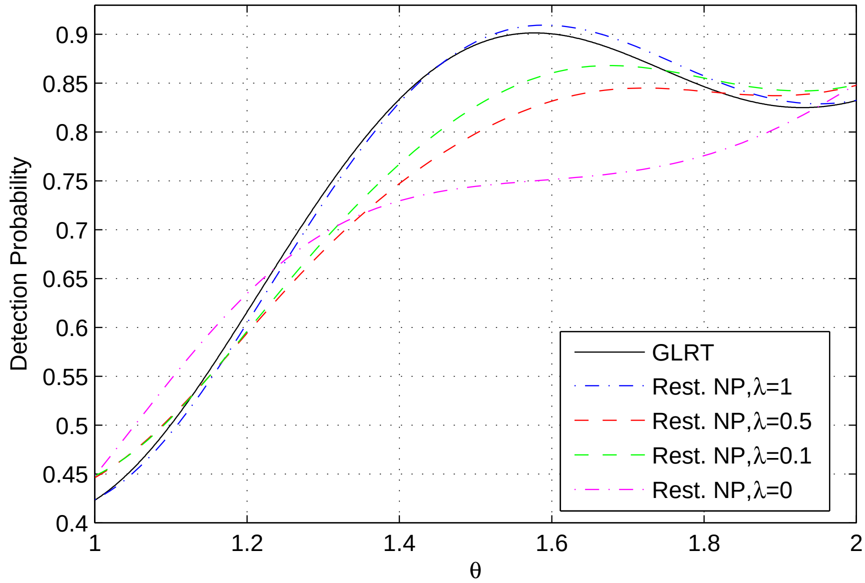


Figure 4.9: Detection probability versus θ for the restricted NP and GLRT approaches, where the parameters of the Gaussian mixture noise are set to $N_m = 3$, $\nu_1 = \nu_3 = 0.25$, $\nu_2 = 0.5$, $\mu_1 = -\mu_3 = 1$, $\mu_2 = 0$, and $\epsilon_i = 0.2\ \forall\ i$.

For the second Gaussian mixture noise parameter set, which is $N_m = 3$, $\nu_1 = \nu_3 = 0.25$, $\nu_2 = 0.5$, $\mu_1 = -\mu_3 = 1$, $\mu_2 = 0$, and $\epsilon_i = 0.2\ \forall\ i$, simulation results for detection probability versus θ and λ are presented in Figure 4.9 and Figure 4.10, respectively. Similar tradeoffs as in the previous scenario are observed. The importance of this scenario is that this kind of mixture noise can be encountered in practice. For example, it can correspond to the sum of zero-mean Gaussian noise and interference which is due to two users that result in signal values of ± 0.5 with equal probabilities at the receiver.

In the third simulations, the parameters of the Gaussian mixture noise are set to $N_m = 2$, $\nu_1 = \nu_2 = 0.5$, $\mu_1 = \mu_2 = 0$, $\epsilon_1 = 0.5$, and $\epsilon_2 = 2$. The noise model employed in this scenario is practically important. The mixture of zero-mean Gaussian random variables with different variances is often used to model man-made noise, impulsive phenomena, and certain sorts of ultra-wideband interference [45]. Figure 4.11 and Figure 4.12 shows the detection probability versus θ and λ for this scenario, respectively. Unlike the previous two cases, the restricted

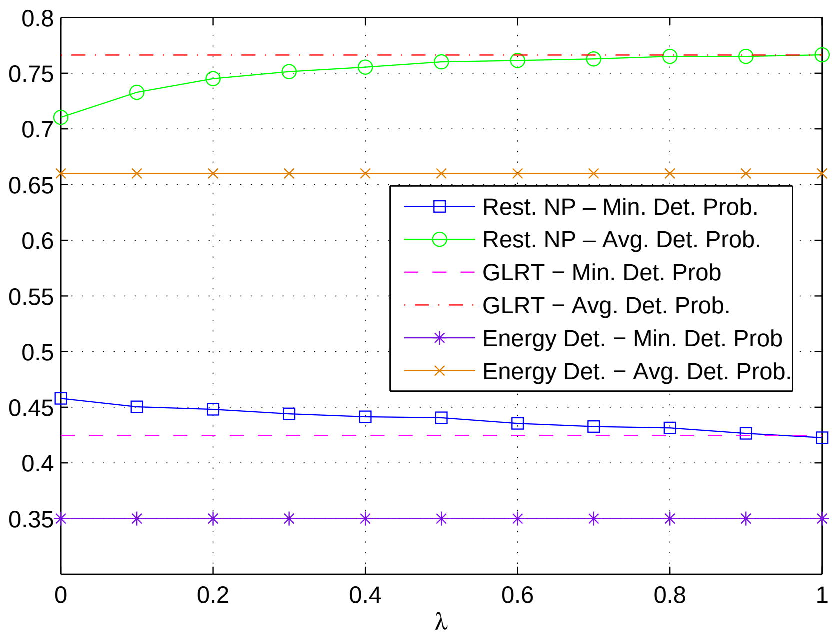


Figure 4.10: Detection probability versus λ for the restricted NP, GLRT and ED approaches, where the parameters of the Gaussian mixture noise are set to $N_m = 3$, $\nu_1 = \nu_3 = 0.25$, $\nu_2 = 0.5$, $\mu_1 = -\mu_3 = 1$, $\mu_2 = 0$, and $\epsilon_i = 0.2\ \forall\ i$.

NP approach can perform better than the GLRT approach in terms of both the minimum and average detection probabilities in this case. After approximately $\lambda = 0.55$, both the minimum and average detection probabilities of the restricted NP approach are larger than that of the GLRT approach. For instance, although the average detection probability of the restricted NP approach is slightly larger than that of GLRT for $\lambda = 0.6$, its minimum detection probability (0.353) is significantly larger than that of the GLRT (0.293).

Based on observations until now in this section, we can conclude that the restricted NP approach can provide important advantages in terms of the worst-case detection probability, and sometimes in terms of the average detection probability (see Figure 2.13) depending on the situation in the presence of imperfect prior information. Until now it is assumed that there is only single observation available to the secondary users. In the following section, simulation results for the multiple observations case are presented.

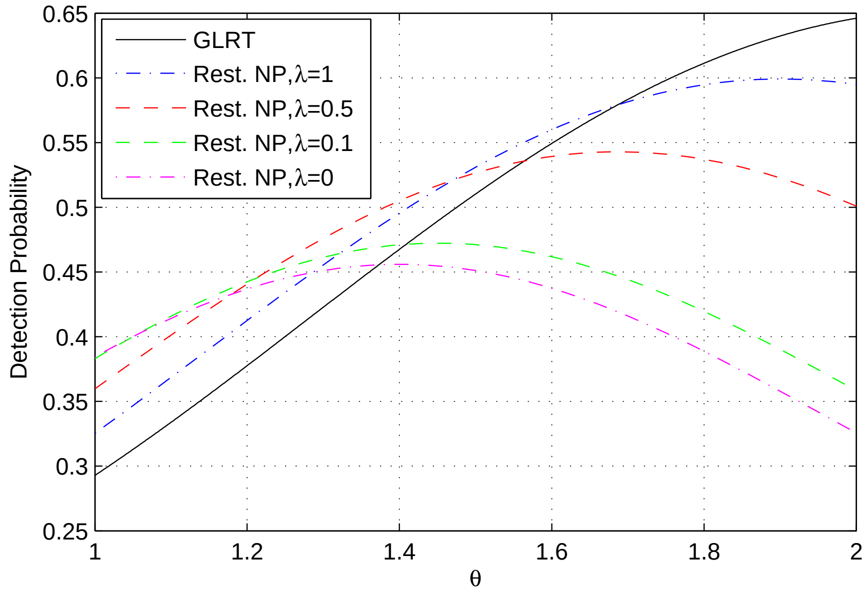


Figure 4.11: Detection probability versus θ for the restricted NP and GLRT approaches, where the parameters of the Gaussian mixture noise are set to $N_m = 2$, $\nu_1 = \nu_2 = 0.5$, $\mu_1 = \mu_2 = 0$, $\epsilon_1 = 0.5$, and $\epsilon_2 = 2$.

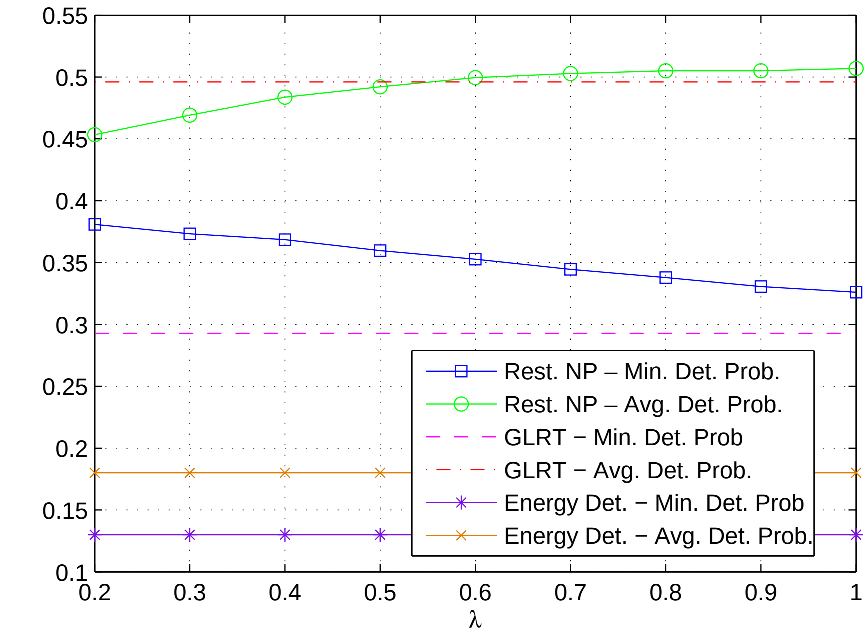


Figure 4.12: Detection probability versus λ for the restricted NP, GLRT and ED approaches, where the parameters of the Gaussian mixture noise are set to $N_m = 2$, $\nu_1 = \nu_2 = 0.5$, $\mu_1 = \mu_2 = 0$, $\epsilon_1 = 0.5$, and $\epsilon_2 = 2$.

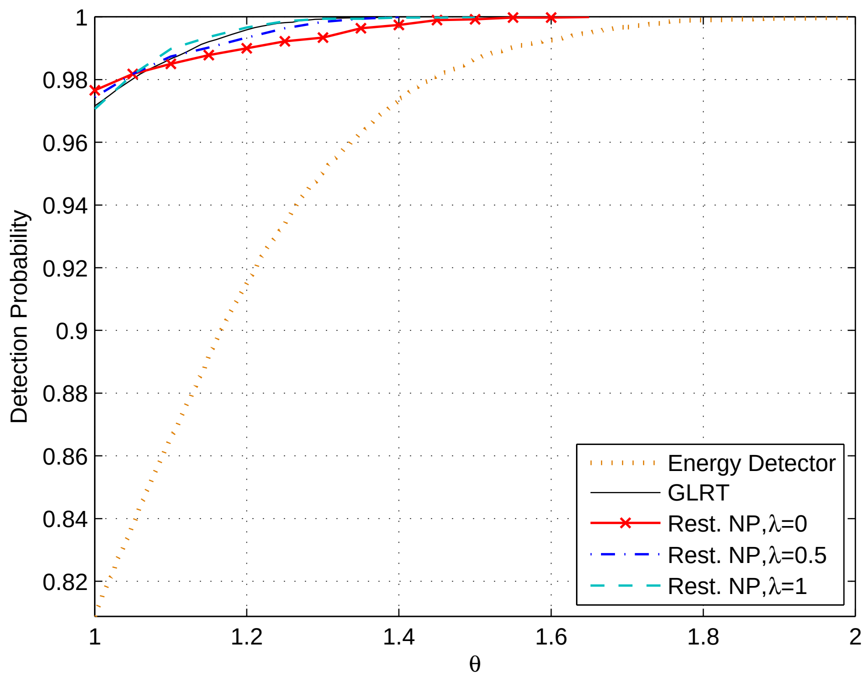


Figure 4.13: Detection probability versus θ for the restricted NP, GLRT, and energy detection approaches for M = 5.

4.2.2 Multiple Observations Case

For the simulations of the multiple observations case, the same distribution for θ and the same value for α are used. Also, the Gaussian mixture noise parameters are set to $N_m = 3$, $\nu_1 = \nu_3 = 0.25$, $\nu_2 = 0.5$, $\mu_1 = -\mu_3 = 1$, $\mu_2 = 0$, and $\epsilon_i = 0.2$. First, in Figure 4.13 detection probability versus θ is plotted for all approaches when $M = 5$ observations are made. It is observed that the GLRT and the restricted NP approaches perform much better than the energy detection. Also, the restricted NP approach provides some improvements over GLRT in terms of minimum detection and average detection performance for various λ values. This can be seen more clearly from Figure 4.14, in which the minimum and average detection probabilities are plotted versus λ. A good tradeoff between the minimum and average detection probabilities can be obtained by employing the restricted NP approach.

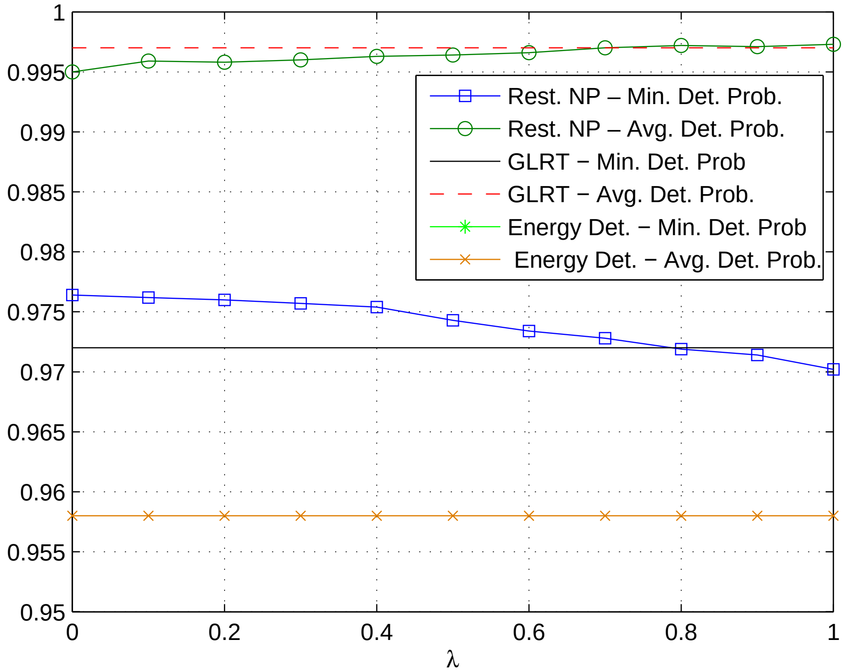


Figure 4.14: Detection probability versus λ for the restricted NP, GLRT, and energy detection approaches for M = 5. For the energy detector, the minimum detection probability is 0.808, which is not shown in the figure.

The performance of the energy detector and GLRT is also investigated for various numbers of observations. Figure 4.15 and 4.16 show that the performance of both GLRT and energy detector improves with the increasing number of observations. Also, GLRT performs significantly better than energy detector for the same number of observations. Since the restricted NP approach performs similarly to GLRT with improved minimum detection or average detection probability depending on the value of λ, we can infer that Figure 4.15 also provides information about the performance of the restricted NP approach.

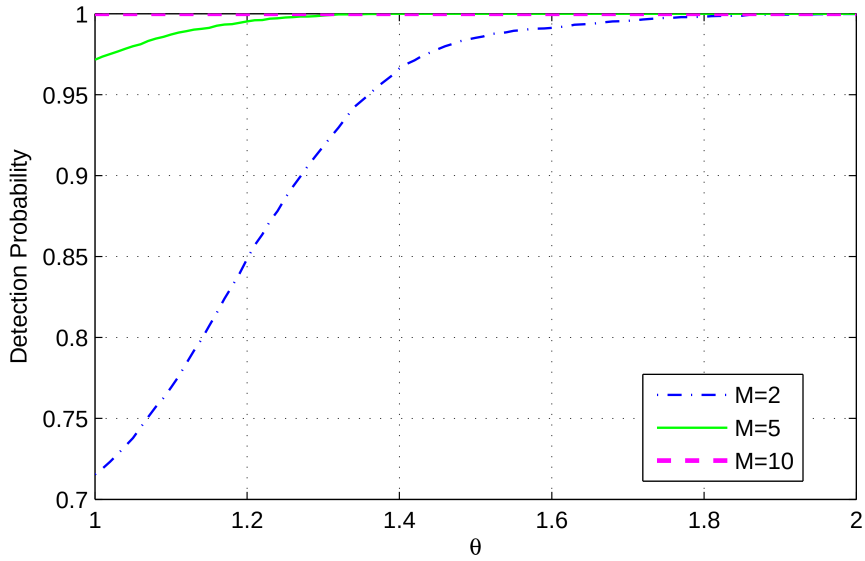


Figure 4.15: Detection probability versus θ for the GLRT approach for various numbers of observations.

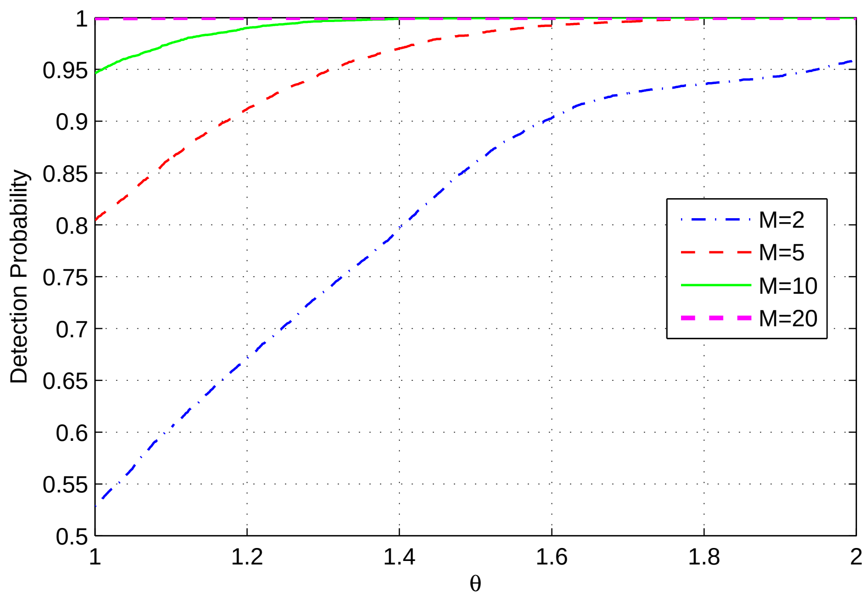


Figure 4.16: Detection probability versus θ for the energy detection approach for various numbers of observations.

Chapter 5

CONCLUSIONS

In this thesis, we have studied the spectrum sensing problem for cognitive radio systems in the presence of uncertainty in the prior distribution of primary users' signal. In Chapter 1, we have first provided background information about cognitive radio and the spectrum sensing problem in cognitive radio systems. Then, the general spectrum sensing problem has been formulated. In addition, three popular spectrum sensing methods; namely, energy detection, matched filtering, and cyclostationary detection have been explained.

In Chapter 2, background information about common hypothesis testing techniques has been provided. We have emphasized the necessity for techniques which take partial information into account. Therefore, we have presented the restricted NP approach [27], an application of the restricted Bayes approach to the NP framework. The problem formulation for the restricted NP approach has been provided, and the algorithm for obtaining the optimal decision rule has been presented.

In Chapter 3, we have investigated how the restricted NP approach can be employed to solve spectrum sensing problem in cognitive radio. We know that some information about the prior distribution of primary users' signal is available in many cases, but that information is not perfect most of the time. This makes the restricted NP approach well suited for our problem. First, we consider Gaussian noise for both Gaussian primary signals and for any kind of primary signals. The simulations in Chapter 4 have shown that although the restricted NP approach outperforms energy detection, it does not provide any significant improvements over the existing GLRT approach. This forced us to consider different scenarios, hence we have studied non-Gaussian noise for the same problem. Particularly, Gaussian mixture noise has been considered, and the simulation results have been presented in Chapter 4. The results have shown that the restricted NP approach performed better than GLRT in terms of the minimum detection probability for different uncertainty levels. Also, in some scenarios (e.g., see Fig. 4.11 and 4.12) the restricted NP approach has higher average detection probability than GLRT after a certain λ value and still it has significantly larger minimum detection probability than GLRT for all λ values. Additionally, we have performed simulations for the case of multiple observations. The simulation results have shown us that performance of GLRT improves with the increasing number of observations which means the performance of the restricted NP also improves because it exhibits similar trends to GLRT with improved minimum detection and/or average detection probability depending on the value of λ.

As a future work, cooperation among cognitive radios which use the restricted NP approach as spectrum sensing method can be employed to compensate the effects of shadowing and multipath fading, and as a result to obtain improved sensing performance.

APPENDIX A

Calculation of Mean and Variance of X

Under hypothesis $\mathcal{H}_1$, mean of $X = \frac{1}{N}\sum_{i=1}^{N}|x_i|^2$ can be found as follows:

$$\begin{aligned}
E\{X\} &= \frac{1}{N}E\left\{\sum_{i=1}^{N}|x_i|^2\right\} \\
&= \frac{1}{N}\sum_{i=1}^{N}E\left\{|s_i+n_i|^2\right\} \\
&= \frac{1}{N}\sum_{i=1}^{N}\left(\underbrace{E\left\{|s_i|^2\right\}}_{\sigma_s^2}+\underbrace{E\left\{s_i n_i^*\right\}}_{0}+\underbrace{E\left\{n_i s_i^*\right\}}_{0}+\underbrace{E\left\{|n_i|^2\right\}}_{\sigma_n^2}\right) \\
&= \sigma_s^2+\sigma_n^2. \qquad \text{(A.1)}
\end{aligned}$$

Since PS users are absent under hypothesis $\mathcal{H}_0$, $E\{X\}$ is equal to σ_n^2 under $\mathcal{H}_0$.

Under hypothesis $\mathcal{H}_1$, $Var(X)$ can be obtained as follows:

$$\begin{aligned}
Var(X) \quad &= E\left\{|X - E\left\{X\right\}|^2\right\} \\
&= E\left\{(\frac{1}{N}\sum_{i=1}^{N}|s_i + n_i|^2 - (\sigma_s^2 + \sigma_n^2))^2\right\} \\
&= \frac{1}{N}E\left\{(|s|^2 + |n|^2 + sn^* + s^*n - \sigma_s^2 - \sigma_n^2)^2\right\} \\
&= \frac{1}{N}E\left\{|s|^4 + 2|s|^2|n|^2 + |n|^4 + (sn^*)^2 + 2(sn^*)(s^*n) + (s^*n)^2\right\} \\
&+ \frac{1}{N}E\left\{2|s|^2(sn^* + s^*n) + 2|n|^2(sn^* + s^*n) + (\sigma_s^2 + \sigma_n^2)^2 - 2|s|^2(\sigma_s^2 + \sigma_n^2)\right\} \\
&- \frac{1}{N}E\left\{2|n|^2(\sigma_s^2 + \sigma_n^2) + 2sn^*(\sigma_s^2 + \sigma_n^2) + 2s^*n(\sigma_s^2 + \sigma_n^2)\right\} \\
&= \frac{1}{N}E\left\{|s|^4 + 4|s|^2|n|^2 + |n|^4 + (\sigma_s^2 + \sigma_n^2)^2 - 2\sigma_s^4 - 4\sigma_s^2\sigma_n^2\right\} \\
&= \frac{1}{N}(E\left\{|s|^4\right\} + 2\sigma_s^2\sigma_n^2 + \sigma_n^4 - \sigma_s^4) \\
&= \frac{1}{N}(E\left\{|s|^4\right\} + 2\sigma_n^4 - (\sigma_s^2 - \sigma_n^2)^2) \qquad \text{(A.2)}
\end{aligned}$$

Since both s_i and n_i have circularly symmetric distribution with zero-mean, we have $E\left\{s^2\right\} = E\left\{(s^*)^2\right\} = 0$ and $\mathrm{E}\{n^2\} = E\left\{(n^*)^2\right\} = 0$. Also, $E\left\{|n|^4\right\} = 2\sigma_n^4$ due to the Gaussian distribution assumption. In short, using circular symmetricity, zero mean, independency of s and n, and the Gaussian n assumptions we obtain $Var(X)$. Similar to the mean, the variance of X is equal to σ_n^4/N under hypothesis $\mathcal{H}_0$ because no PS users are present under $\mathcal{H}_0$.

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